Validation of Empirical Yield Curves For Ontario

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Abstract

In 2000, the Forestry Research Partnership initiated the Benchmark Yield Curve Project to provide growth and yield estimates for forest management planning. The resulting yield curves incorporate the more than 3,000 permanent sample plots (PSPs) maintained in Ontario as well as PSPs from neighbouring jurisdictions. Equations were fit to the data to predict the growth and yield of the Northeast Region standard forest units for the use in wood supply estimation. A similar set of equations were fit to the data by leading species.

The results are validated against recently collected data and compared to predictions from Plonski's yield table, modified Plonski, and Northeast regional curves, the current wood supply yield curves in Ontario. The results of the validation show the new yield curves generally have lower prediction errors for gross total volume than Plonski and modified Plonski with the exception of the MW2 and SF1 forest units. In contrast to the other forest units, these forest units are more mixed in terms of species, light tolerance and ages. For the MW2 and SF1 forest units, a leading species approach is recommended. Similar results were observed for net merchantable volume.

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> *Most modeling efforts commence with any data available, and the modeling approach often may be dictated by limitations of the data. Many models owe much of their success to the foresight and dedication of our forebears who established permanent plots and maintained both plots and measurement records carefully. (Vanclay 1994, p79).*

1 Objective

The purpose of this project is

- to test recently developed empirical yield curves against independent data,
- to compare the forest unit and leading species approaches to yield curve development, and
- to compare the precision and accuracy of the new yield curves against currently available yield prediction tools in Ontario.

This project is part of a larger initiative to develop yield curves based on field data for the analysis of wood supply. These curves should represent a range of current and potential management intensities for the standard northeast forest units (Watt et al. 2001) and southcentral (draft) forest units. The curves should be compatible with the sustainable forest management model (SFMM) (Davis 1999) and be applicable to forest management units (FMUs) within Tembec's operating area in Ontario. The intent of the project was to use all available growth and yield plot data.

2 Background

The Ontario Forest Accord (OMNR 1999) outlined 31 commitments agreed to by members of the forest industry, the Partnership for Public Lands, and the Ontario Ministry of Natural Resources. Commitment 5 commits to the development of an Ontario forest science partnership, in part to assess the impacts of intensive forest management on increased forest growth and yield. Tembec's response to this commitment led to the creation of the Forestry Research Partnership (FRP) that includes the Canadian Forest Service, the Ontario Ministry of Natural Resources and Tembec.

In an Intensive Forest Management (IFM) workshop in Sault Ste. Marie in 1999, participants noted the following needs (Bell et al. 2000, p31).

- Locally calibrated (region-, subregion-, and FMU-specific) yield curves for the full spectrum of silvicultural treatment options. These curves should be based on forest units and ecosites.
- Yield curves for mixedwood stands.
- Yield curves for managed stands including plantations established with improved stock and/or effective vegetation management.
- Yield curves for partial harvesting and thinning regimes.

Participants also identified the need for objective, peer-reviewed programs and projects in order to gain reliable information.

Yield curves are an integral part of forest management planning. Although the definitions of the standard forest units have been agreed upon, the forest inventory attributes of a forest unit vary from management unit to management unit depending on the soil types, past disturbances and past management. Therefore, the yield curves will vary with management unit.

2.1 The forest units

This project uses the standard forest units for the Northeast Region (Watt et al. 2001). In the forest management planning manual (OMNR 2004) a forest unit is defined as "an aggregation of forest stands for management purposes which have similar species composition, develop in a similar manner, (both naturally and in response to silvicultural treatments) and are managed under the same silvilcultural system."

2.2 Existing curves

The curves currently used in most forest management plans are derived from the default yield curves in SFMMTool (Watkins 2004) which are derived from Plonski's (1981) yield tables. Northeast Regional curves (by Neil Maurer) and custom or user-defined curves are also available. Plonski's (1981) curves are based on temporary sample plots with the bulk of the data collected prior to 1960. The tables were modified to project yields for older stands and to also reflect mortality following rotation age. Maurer's curves were also based on temporary sample plots, primarily in natural, untreated stands.

It was anticipated that developing curves from repeated measurements on permanent sample plots and using all the plot data available in Ontario would provide improved yield estimates and, most importantly, provide empirical estimates of growth.

3 DATA

Plot data are summarized in Table 1. The data come from the Ontario provincial database as well as the Canadian Forest Service and the Quebec Forest Service. Raw data were compiled to a standard form described elsewhere[1].

3.1 Representativeness of the data

The data analysis focuses on ensuring the model forms fit the data. It is also essential that the data reflect the population. It is not necessary that the plots be a random sample from the population (Iles 2003). In general, historic plots like the AmericanCan and Kimberly Clark plots were located on better sites with high stocking. They tend to represent better than average conditions. Experimental sites such as the Thunder Bay spacing trial and those from the Petawawa Research Forest tend to have above average growth due to complete site occupancy, above average protection, and better tending. It is important that these attributes be included as covariates in the model to avoid bias. The more recently established growth and yield plots (1994+) are more representative of the range of sites in the population.

The Beckwith Roebbelen plots at Limestone Lake were removed because they occur on limestone soils, not representative of Tembec's license areas, and have significantly higher growth.

The Quebec plots are a nested design. Small trees are measured on a small plot and the density (stems/ha) estimates had high variability. In general, the Quebec plots were not used to estimate the density functions.

The observations in this study are not independent. Some plots have as many as 8 observations. Some plots are located in the same stand. Repeated measurements on the same plots tend to be correlated as are measurements from plots that are close to one another. When ordinary least squares techniques are used with such data, the parameter estimates are unbiased, but the covariance matrix associated with the parameter estimates and the equation variances may be underestimated (Vanclay 1994).

4 Methods

In order to be useful in wood supply modeling, yield curves must be compatible with forest resources inventory (FRI) data. The FRI data for each stand consist of the species composition (to the nearest 10%), the age class, stocking, average height of dominant and codominant trees, and site class. Various other classification variables are also available including ownership, site region, and site district. Generally, silvicultural history is not available as part of the FRI. The yield equations were fit by forest unit and origin (natural vs. planted stands).

According to Vanclay (1994 p.18), yield equations assume a prescribed management regime. Growth equations have the advantage that silvicultural treatments such as thinning and spacing can be simulated at any time. Yield equations should generally be appropriate for natural and extensive conditions with no management intervention. Yield equations may also be used to predict growth under unmanaged conditions assuming that stocking is constant over time.

Data compilation and analysis were conducted using SAS® BASE and STAT statistical software. The graphs were produced by putting the data and the equations into Microsoft Excel.

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[1] http://www.forestresearch.ca/product_catalogue/Boreal_Feb04.pdf

The potential independent variables were age, site index, and stocking – attributes available from the inventory.   Forest unit can be derived from the species composition in the inventory and stand origin should be available from silvicultural records so these attributes were also included.

4.1 Basal area

For even-aged forests, basal area increases with site index and stocking. Basal area also increases with age with a rapid increase at young ages that slows as the stand achieves full site occupancy. Basal area was predicted as a linear function of stocking and site class and a sigmoidal function of age using the following equation form.

$$(1) \qquad \hat{B}A = stocking \cdot (\alpha_0 + \alpha_1 \cdot \hat{S}I) \cdot (1 - e^{-\beta_0 \cdot age^{\gamma_0}})$$

The $stocking \cdot (a_0 + a_1 \cdot SI)$ term represents the upper asymptote of basal area, the maximum basal area that a stand with that stocking and SI can achieve. The remainder of the equation predicts how rapidly the basal area approaches that maximum.

Equation (1) should be a relatively good predictor of basal area since the stocking is the ratio between actual and theoretical (Plonski) basal area. In the FRI the stocking is generally estimated from aerial photography, not calculated, and the accuracy and precision of that estimate is unknown.

4.2 Top height and site index

Top height curves are an essential of part of the yield curves developed here. Historically, height data have been expensive to collect and were highly variable and generally deficient for minor species. Top height curves were taken from the literature and evaluated against the observed height development patterns in the data. Uncertainties and errors in the site index curves are not included in the yield prediction intervals.

Top height for jack pine was predicted using the following equation from Carmean (2001) equation 3.

$$top\hat{h}t = 1.3 + 4.1459 \cdot (\hat{S}I - 1.3)^{0.6224} \cdot (1 - K^{(age - age2bh)/50})^{1.3723 \cdot (\hat{S}I - 1.3)^{-0.0802}}$$

(2) where

$$K = 1 - \left(\frac{\hat{S}I - 1.3}{4.1459 \cdot (\hat{S}I - 1.3)^{0.6224}}\right)^{\frac{1}{1.3723 \cdot (\hat{S}I - 1.3)^{--0.0802}}} \quad \text{and } age2bh = \text{age to breast height}$$

Top heights for the remaining species were predicted using equations from appendix III and IV of Carmean (1996). The equations associated with figure 11 of Carmean (1996) were used for black spruce. The equations associated with figure 17 of Carmean (1996) were used for white spruce (note that the white spruce plantation curves in Carmean (1996) were not used). The equations associated with figure 18 of Carmean (1996) were used for balsam fir. For aspen, the equations from Carmean et al. (2002) were used. Site index for aspen was determined using the same equation and a search algorithm. The predictions from the equation for aspen in Carmean et al. (2002) are similar to those in Carmean (1996) up to approximately age 90 but then flatten out more quickly than Carmean (1996).

The age to reach breast height (*age2bh*) was assumed to be 6 years for natural stands and 5 years for plantations.

4.3 Density

Density changes with stocking, age, site index, and basal area. Several variations were tried including raising the independent variables to negative exponents and using the inverse of the independent variables. When comparing the alternative equations, particular attention was paid to the model behaviour at older ages.

The following equation form provided good predictions for density (stems/ha).

(3) $\hat{stems} = \frac{x_0}{SI} \cdot stocking^{-x_1} \cdot \hat{BA} \cdot age^{-x_2}$

The variance of the residuals increased with predicted stems so the observations were weighted by $\frac{1}{\hat{BA}}$ resulting in a more homogenous variance for the residuals.

4.4 Volume

The following equation form was used to predict total stem volume (m^3/ha). The coefficient is analogous to the cylindrical form factor $f = \frac{vol}{BA \cdot ht}$ (Husch et al. 1972).

(4) $\hat{vol} = z_0 \cdot \hat{BA} \cdot top\hat{ht}$

The variance of the residuals increased with predicted volume so the observations were weighted by $\frac{1}{\hat{BA}}$ resulting in a more homogenous variance for the residuals.

Merchantable volumes were calculated using the scaling manual (OMNR 1995) minimum standards. That is, a stump height of 30 cm and a top diameter of 16cm (white & red pine, hemlock, poplar, or white birch), 10 cm (other conifer), or 20 cm (other hardwood). Initially merchantable volume was predicted as a simple proportion of the total volume. In order to capture the increase in the proportion of volume that is merchantable, an age term was added.

(5) $m\hat{vol} = z_3 \cdot (1\text{-}e^{-z_1 * age^{z_2}}) \cdot \hat{vol}$

The relationship between net merchantable volume and age was obtained by fitting the following equation to the cull factors from OMNR (1978), by species. This predicts the cull fraction increases as a sigmoidal function of age.

(6) $\hat{cull}_i = (1 - e^{-d_0 \cdot age})^{d_1}$

For each species, the relationship between the volume (GTV) to basal area ratio (VBAR) and age was estimated using the entire data set.

For each species, the net merchantable volume was estimated using the following equation.

(7) $nm\hat{vol}_i = \frac{sppba_i \cdot \hat{cull}_i \cdot \hat{vbar}_i}{\sum_i sppba_i \cdot \hat{cull}_i \cdot \hat{vbar}_i} \cdot m\hat{vol}$ where $sppba_i$ is the basal area for species i

4.5 Mixed species forest units

Some of the forest units do not have a single dominant species. For example, in the Romeo Malette Forest, the average species composition for the MW1 forest unit is $Pj_{34}Po_{31}Bw_{19}Sb_{11}Sw_2Bf_2$. In the boreal forest region of Ontario, the MW1, MW2 and SF1 are the most commercially important mixed forest units. If the MW1 is broken down by leading species, approximately 40% of the area in the MW1 forest unit has jack pine leading, 34% poplar, 20% white birch with minor areas in balsam fir, cedar, and black spruce. Generally the leading species is at least 40% of the species composition. In deriving the summary of the forest unit, SFMMTool averages the site classes, regardless of leading species, and assigns the average site class to the leading species of the average species composition.

As an artificial example, let half the forest unit be $Pj_{60}Po_{40}$ with a site class of 2 and the other half be $Po_{60}Pj_{40}$ with a site class of 1. The average species composition is $Pj_{50}Po_{50}$ with an average site class of 1.5.

If the average site class is applied to the jack pine, it implies a dominant height of 16.8m at age 50. If the average site class is applied to poplar, it implies a dominant height of 21.6m at age 50.

Likewise stocking is dependent on the leading species.

The situation is more complicated because the site class is not necessarily based on the leading species of the stand. Where possible, future inventories should record the species for which the age, height, and site index were computed.

For forest units that have more than one leading species represented by significant areas, the following approach is recommended. It is based on the assumption that the age, height, site index, and stocking attributes are based on the leading species. As noted in the previous paragraph, this is not always the case. However, it is beyond the scope of this project to solve that problem.

1. Calculate the forest unit summaries by leading species. Leading species with minor areas (<5% of the forest unit area or less than 100ha) may be combined with other leading species.
2. Derive yield curves for each leading species using the yield curves derived for the forest units with a single dominant species. For the artificial example above, a yield curve for the $Pj_{60}Po_{40}$ area would be derived from the PJ1 or PJ2 forest unit (as appropriate) and the yield curve for the $Po_{60}Pj_{40}$ portion derived from the PO1 curves. The two yield curves would then be averaged (weighted by area) to produce the mixed forest unit yield curve.

There are a number of advantages to this approach. It attempts to keep site index associated with a species. It also allows the leading species to be used when specifying management intensities. For example, for spruce/aspen mixes, an intensive forest management strategy may involve removing the aspen overstory at age 60 and leaving the spruce for another 40 years before final removal, whereas intensive management in the jack pine/aspen mixes may involve early cleaning to remove the aspen and rehabilitate the site to a pine forest unit. In either case, the leading species is important in determining the silvicultural prescription, not the average leading species.

4.6 Leading species

The previous analysis was based on forest unit and origin. A similar fitting procedure was undertaken by leading species rather than forest unit. The leading species is the species with the highest percentage of basal area on the plot.

5 Results

The coefficients associated with models (1), (3), (4) and (5) are given in Table 2 through Table 5. The tables also indicate the average prediction error as well as the mean squared error.

The average error of the basal area predictions was less than 3% for all forest units except the MW2 forest unit. The average error of the density predictions was less than 4% for all forest units except the BW1 (5.6%) and PJ1 (6.0%). The average error of the gross total volume predictions was less than 4% for all forest units except the LH1 (6.7%), MW2 (18.9%) and the SF1 (12.2%).

6 Validation and comparison to other models

How good is the growth and yield model? One method of evaluating a model is validation.

> Validation is a demonstration that a model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model. Rykiel (1996).

The provincial PSPs, PGPs and the Growth and Yield Coop PSPs were not included in the model development but were reserved for validation.

Natural origin plots, older than 20 years and with a stocking of greater than 0.30 were used for validation. The validation data set is summarized in Table 1.

The predictions were compared against the most common yield prediction models used for forest management planning in Ontario – Plonski, modified Plonski and Northeast regional. The Plonski, modified Plonski and Northeast regional predictions were obtained by running the plot summaries through the volume calculator of SFMMTool (Watkins 2004).

The Plonski yield curves (Plonski 1981) are the most recently published yield curves for Ontario but the number of species represented is limited and the yield curves generally end at around age 100 for intolerant species. Density, basal area, gross total volume and gross merchantable volume are given in Plonski (1981).

The modified Plonski predictions are available in SFMMTool but have little documentation. Essentially, modified Plonski adds net merchantable volume and extends the original Plonski yield curves to age 250, generally by predicting a decrease in net merchantable volume to zero by approximately age 160. White and black spruce show a much slower drop in volume and volumes for tolerant hardwoods, hemlock, and white and red pine are constant past 150 years. Gross total volume, net merchantable volume, current annual total volume increment, and periodic annual total volume increment are available in SFMMTool for modified Plonski.

The Northeast regional curves are generally similar to modified Plonski except show a sharper decline in net merchantable volume in jack pine, white spruce, poplar, and white birch with age. Only net merchantable volume is available in SFMMTool for the Northeast regional curves.

A comparison of the gross total volume predictions for the validation dataset is given in Figure 2. In general, Plonski and modified Plonski yield curves overpredicted volume while the forest unit and leading species approached generally underpredict volume. In general, the forest unit approach had the lowest average prediction error for gross total volume with the notable exception of the MW2 and SF1 forest units. These forest units are mixed in terms of species composition, light tolerance, and ages. The MW1 is also a mixed species forest unit but tends to be more even-aged. For the MW2 and SF1 forest units, where the forest unit approach performed poorly, the leading species predictions were considerably more accurate.

A comparison of the net total volume predictions for the validation dataset is given in Figure 3. In general, the average prediction errors for net merchantable volume were smaller than for gross total volume due in part to lower volumes. The Northeast regional and Plonski modified curves overpredicted the hardwood and spruce dominated forest units. The forest unit approach generally had the lowest mean prediction errors except for the MW2 and SF1 forest units. Although the leading species approach improved the estimates for the MW2 and SF1 forest units, the improvement was not as dramatic as for gross total volume.

7 Conclusion

The yield curves developed for the Northeast standard forest units are based on substantially more data than existing yield curves for Ontario (Plonski, modified Plonski and Northeast regional curves). They were also developed specifically for the Northeast standard forest units. When used to predict yields for an independent dataset, the new yield curves were generally more accurate and precise than existing yield curves with the exception of the MW2 and SF1 forest units. These two forest units represent a mixture of species, tolerances and ages and yields are not predicted well with the new forest unit curves. For these forest units, a leading species approach is recommended.

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Table 1. The plots are summarized by forest unit. Means are followed by the data ranges in brackets.

FU	Number of plots	Number of measurements	Age (years)	Basal area (m^2/ha)	Density (stems/ha)	Stocking	Site index (m)	Gross total volume (m^3/ha)
Calibration data								
BW1	1141	1725	56	18.5	1374	0.94	17.8	115
			(0-148)	(0-77)	(8-11000)	(0-4.45)	(6-261)	(0-530)
LC1	326	468	80	25.4	1709	0.69	14.0	137
			(0-192)	(0-98)	(25-11125)	(0.01-7.1)	(2-194)	(0-596)
LH1	1233	2321	54	30.1	1996	0.84	19.3	185
			(0-163)	(0-113)	(25-12575)	(0.02-3.87)	(6-90)	(0-852)
MW1	175	303	57	27.1	1735	1.00	17.9	195
			(0-168)	(0-82)	(25-5775)	(0.01-1.82)	(9-35)	(0-644)
MW2	1792	2752	59	27.8	1980	0.86	18.0	173
			(0-214)	(0-102)	(50-13500)	(0-2.00)	(7-247)	(1-869)
PJ1	931	2306	50	21.0	2084	0.94	16.3	144
			(0-152)	(0-65)	(8-27725)	(0-1.99)	(10-35)	(0-466)
PJ2	328	492	66	24.9	1699	0.97	16.2	174
			(0-176)	(0-72)	(33-8125)	(0.01-2.02)	(9-35)	(0-473)
PO1	1018	1705	55	26.7	1473	0.99	21.0	211
			(0-176)	(0-93)	(8-10475)	(0-3.61)	(10-53)	(0-685)
SB1	2558	4888	97	17.3	1708	0.5	10.3	93
			(0-244)	(0-78)	(8-12425)	(0-3.12)	(0-104)	(0-513)
SF1	3630	6229	71	26.1	2131	0.68	15.6	144
			(0-261)	(0-122)	(5-17700)	(0-4.64)	(2-167)	(0-875)
SP1	804	1333	72	23.1	1860	0.61	13.9	143
			0-211)	(0-89)	(50-14225)	(0.02-2.99)	(4-63)	(1-478)
Validation data								
BW1	57	59	59	21.5	1444	1.02	17.1	144
			(23-107)	(9-38)	(450-3075)	(0.52-1.81)	(10-29)	(41-316)
LC1	24	27	96	25.1	2162	0.69	10.7	138
			(48-198)	(10-49)	(1083-4275)	(0.31-1.42)	(5-18)	(54-310)
LH1	3	3	58	19.9	1186	0.84	17.1	130
			(46-73)	(13-25)	(825-1683)	(0.67-1.02)	(16-19)	(79-173)
MW1	61	62	53	22.4	1744	0.93	19.0	155
			(20-120)	(8-35)	(367-4175)	(0.3-1.63)	(11-26)	(44-317)
MW2	146	152	63	23.8	1630	0.87	17.4	161
			(21-144)	(7-48)	(250-4825)	(0.33-1.85)	(10-29)	(35-338)
PJ1	258	282	56	23.4	1723	0.91	18.0	166
			(20-118)	(5-42)	(325-5400)	(0.33-1.61)	(10-26)	(19-339)
PJ2	87	91	62	25.0	1843	0.98	17.9	174
			(20-141)	(8-47)	(375-3975)	(0.33-1.81)	(11-25)	(36-392)
PO1	159	171	57	24.4	1506	0.91	21.3	195
			(20-122)	(5-47)	(225-4817)	(0.33-1.58)	(12-30)	(22-408)
SB1	202	220	89	24.5	2186	0.65	12.5	142
			(23-193)	(9-52)	(575-5600)	(0.3-1.29)	(5-23)	(33-350)
SF1	137	144	59	23.4	1959	0.61	15.3	136
			(21-138)	(6-47)	(275-5050)	(0.31-1.11)	(7-25)	(18-309)
SP1	128	132	70	23.9	1874	0.64	14.2	152
			(22-151)	(7-44)	(317-5375)	(0.3-1.36)	(7-24)	(25-299)

Table 2. The results of fitting the basal area model (equation (1)) are given by forest unit and leading species.

Forest unit or Leading species	a0	a1	b0	c0	Mean error	Mean squared error
BW1	5.3582	1.0792	0.0010	2.0009	0.103	3.747
LC1	31.6596	0.6863	0.0010	1.9531	-0.063	6.622
LH1	19.8588	0.5764	0.0104	1.3059	0.612	18.392
MW1	19.9710	0.4649	0.0279	1.0990	0.370	17.800
MW2	9.4023	0.8555	0.0010	2.1399	3.261	100.635
PJ1	7.9126	1.0731	0.0484	1.0625	0.063	1.665
PJ2	16.4714	0.5376	0.1136	0.8988	0.170	6.029
PO1	11.4107	1.0390	0.0047	1.5366	0.049	1.967
SB1	22.7056	1.4822	0.0020	1.7156	0.121	2.440
SF1	27.0589	0.8168	0.0010	2.0311	0.568	12.821
SP1	30.7539	0.6128	0.0010	2.0703	0.182	9.243
Jack pine	11.5089	0.8393	0.0419	1.1373	0.015	1.487
White spruce	25.3449	1.2558	0.0023	1.7050	0.153	2.850
Black spruce	24.0199	1.3080	0.0010	1.9474	0.078	2.629
Balsam fir	30.4803	0.7386	0.0010	2.0125	0.095	4.099
Cedar	31.0357	0.7977	0.0010	1.9545	0.048	6.880
Larch	40.0000	0.2724	0.0241	1.1039	0.127	2.415
White birch	5.9763	0.9291	0.0010	2.0503	0.664	16.793
Trembling aspen	0.0106	1.4927	0.0015	1.8877	0.807	28.266

Table 3. The results of fitting the stem model (equation (3)) are given by forest unit and leading species.

Forest unit or Leading species	x0	x1	x2	Mean error	Mean squared error
BW1	27378	0.0000	0.7777	80.5	283696
LC1					
LH1	68939	0.3706	1.0337	-20.55	454053
MW1	136650	0.0994	1.1799	1.66	493308
MW2	59964	0.3172	0.9488	-12.65	687757
PJ1	413738	0.1554	1.5126	104.15	886580
PJ2	25552	0.0000	0.7567	-17.76	501155
PO1	345476	0.0310	1.4261	-57.29	343194
SB1	283114	0.2222	1.2882	-34.37	331767
SF1	111885	0.3944	1.2395	-35.57	170304
SP1	125471	0.0000	1.1349	-21.45	232173
Jack pine	316238	0.1360	1.4307	97.92	850339
White spruce	269590	0.0000	1.4324	-19.48	171713
Black spruce	126641	0.0652	1.1314	-36.5	352141
Balsam fir	161105	0.0000	1.2265	-18.24	620134
Cedar	339277	0.0372	1.4494	-33.1	237481
Larch	173764	0.0912	1.2287	-5.05	43508
White birch	89959	0.0022	1.0762	-26.98	396518
Trembling aspen	108159	0.1496	1.1183	-45.63	494353

Table 4. The results of fitting the gross total volume model (equation (4)) are given by forest unit and leading species.

Forest unit or Leading species	z0	Mean error	Mean squared error
BW1	0.3748	1.539	661.3
LC1			
LH1	0.3584	8.743	8.7
MW1	0.3874	5.680	1196.1
MW2	0.3228	30.536	5616.9
PJ1	0.4232	7.029	422.9
PJ2	0.3869	6.382	727.6
PO1	0.3833	0.564	566.8
SB1	0.3671	-0.424	187.6
SF1	0.2834	16.536	3068.3
SP1	0.4148	1.232	777.7
Jack pine	0.4136	5.923	444.4
White spruce	0.3365	3.869	269.6
Black spruce	0.3921	-0.062	400.6
Balsam fir	0.3535	2.199	581.4
Cedar	0.3141	0.000	0.0
Larch	0.3701	4.285	205.4
White birch	0.3359	13.652	2069.5
Trembling aspen	0.3569	16.151	5487.8

Table 5. The results of fitting the gross merchantable volume model (equation (5)) are given by forest unit and leading species.

Forest unit or Leading species	z1	z2	z3	Mean error	Mean squared error
BW1	0.0000	3.8708	0.6019	-1.794	1004.3
LC1					
LH1	0.0024	1.8352	0.3714	1.854	903.1
MW1	0.0031	1.5383	0.7663	1.742	1126.5
MW2	0.0068	1.5453	0.6737	11.836	2550.7
PJ1	0.0072	1.4995	0.8289	-2.204	561.5
PJ2	0.0425	0.9736	0.8067	1.437	1070.6
PO1	0.0000	3.6243	0.8062	-3.849	1849.0
SB1	0.0004	1.9130	0.7930	-2.212	628.1
SF1	30.7765	1.9795	0.7702	17.078	3565.8
SP1	0.0093	1.1688	0.8789	1.045	1188.6
Jack pine	0.0083	1.4702	0.8072	-2.225	776.6
White spruce	0.0172	1.1263	0.8643	-1.944	500.0
Black spruce	0.0082	1.2060	0.8517	-0.916	821.5
Balsam fir	0.1182	0.4799	1.0000	-0.461	627.2
Cedar	0.5565	0.5000	0.5000	7.212	52.0
Larch	1.1243	1.0689	0.7075	13.834	904.1
White birch	0.0032	1.6758	0.5597	3.237	861.3
Trembling aspen	0.0000	3.4585	0.7871	9.021	4033.7

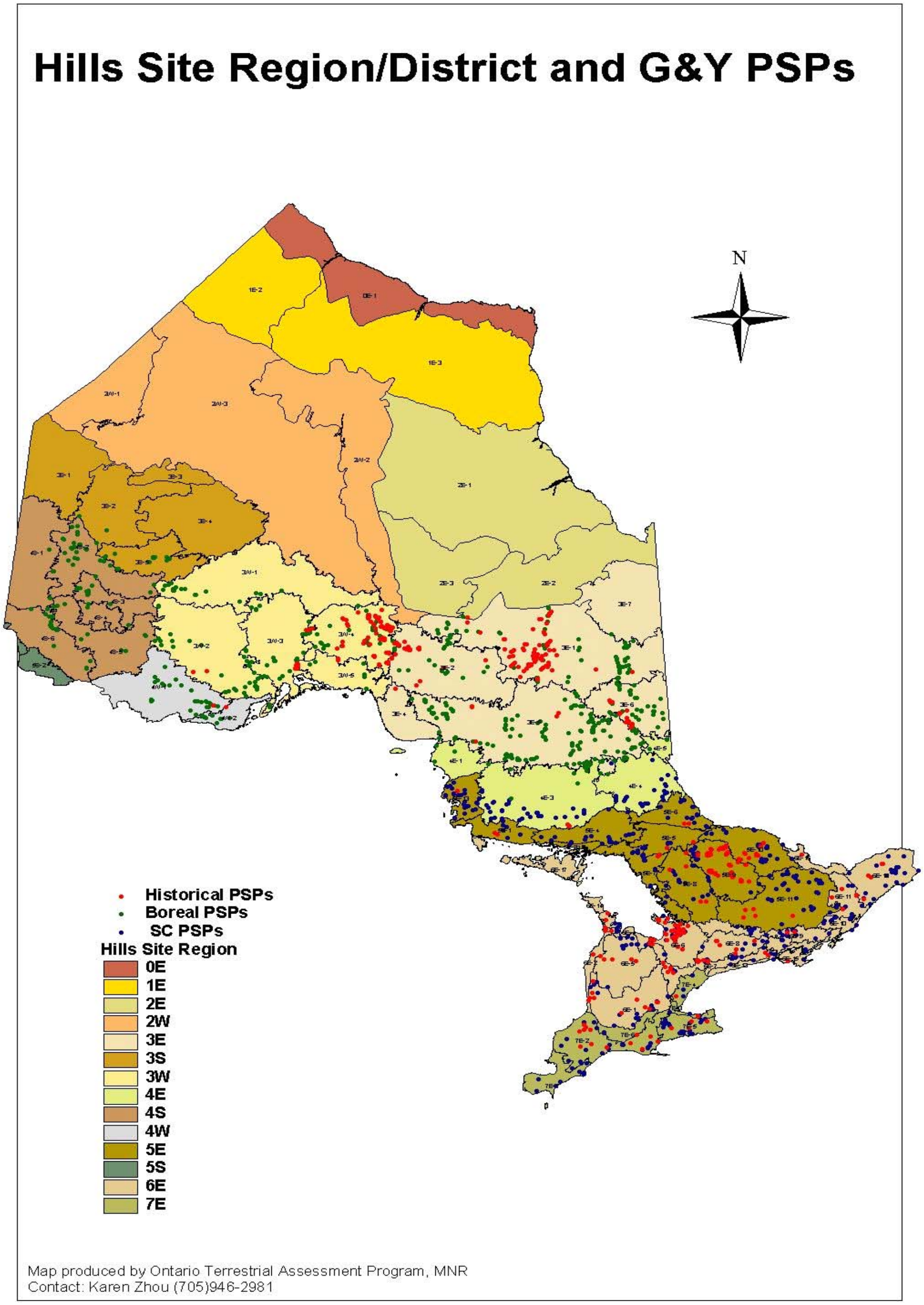
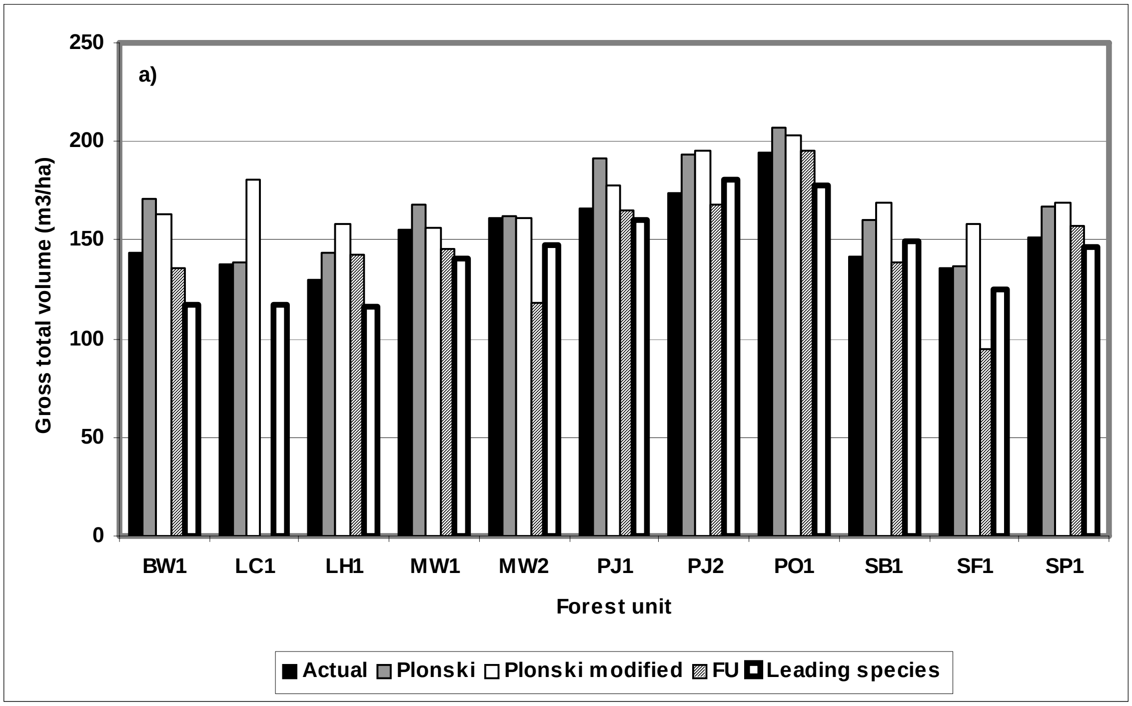


Figure 1. The plot locations are given by Hills site region. “SC” refers to southcentral.



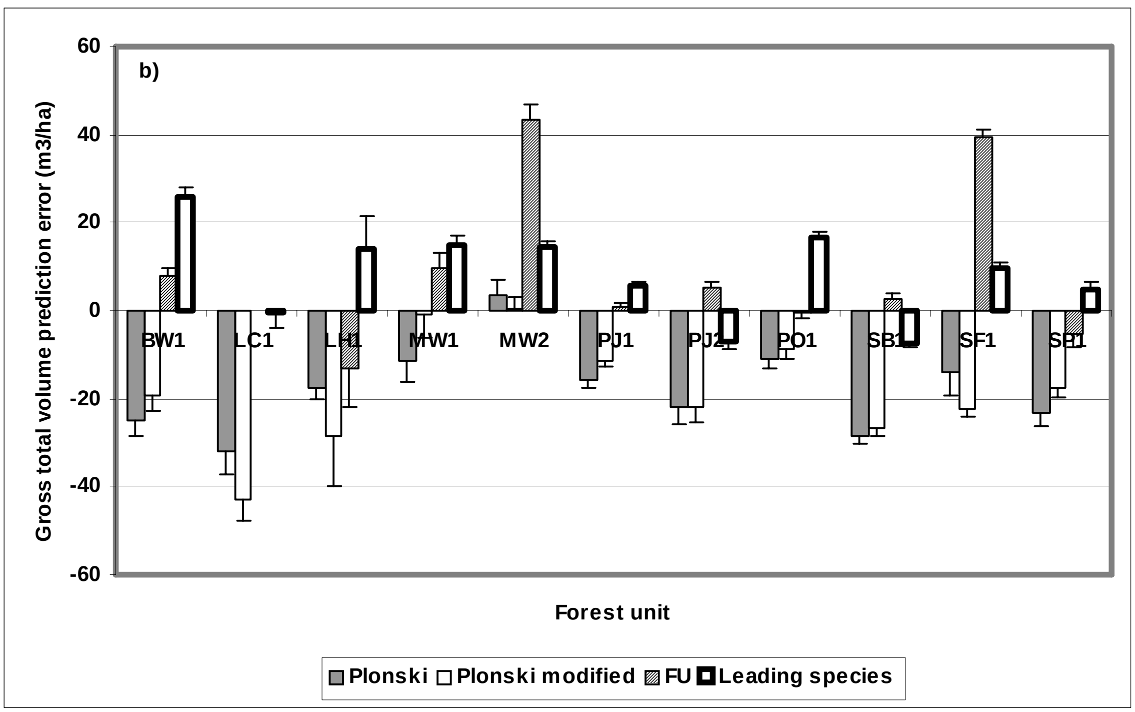


Figure 2. The average actual and predicted gross total volume (a) and the average prediction error (bias) and the standard error of the prediction errors are given by forest unit (b) for the validation data. Gross total volume is not available for the Northeast regional yield curves. A positive prediction error indicates an underprediction.

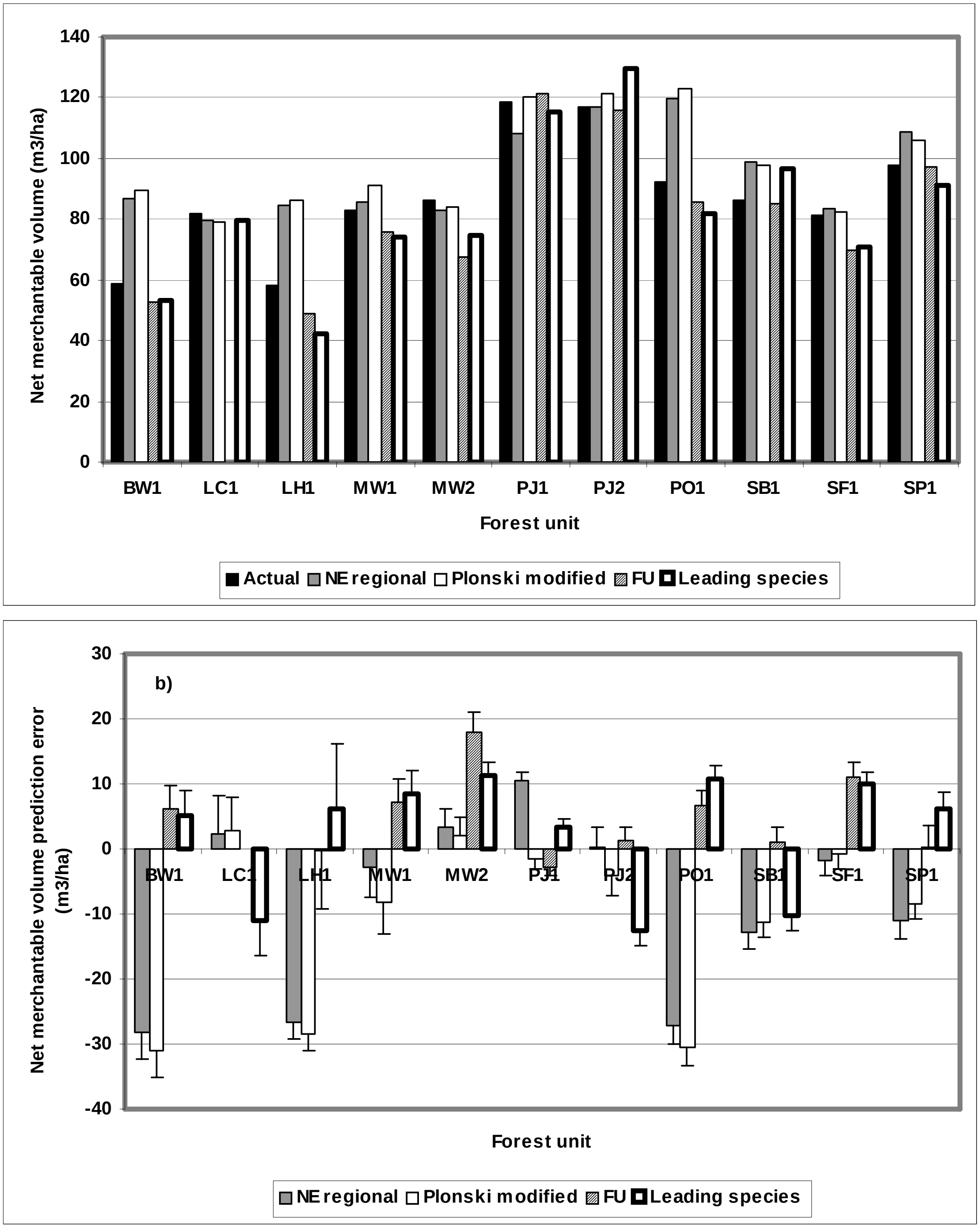


Figure 3. The average actual and predicted net merchantable total volume (a) and the average prediction error (bias) and the standard error of the prediction errors are given by forest unit (b) for the validation data. Net merchantable volume is not available for Plonski's yield curves. A positive prediction error indicates an underprediction.