Predicting forest stand variables from LiDAR data in the Great Lakes – St. Lawrence forest of Ontario

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ABSTRACT

Models were developed to predict forest stand variables for common species of the Great Lakes – St. Lawrence forest of central Ontario, Canada from light detection and ranging (LiDAR) data. Stands that had undergone various ranges of partial harvesting or initial spacing treatments from multiple geographic sites were considered. A broad forest stratification was adopted and consisted of: (i) natural hardwoods; (ii) natural conifers; and (iii) plantation conifers. Stand top height (R^2 = 0.96, 0.98, and 0.98); average height (R^2 = 0.86, 0.76, and 0.98); basal area (R^2 = 0.80, 0.80, and 0.85); volume (R^2 = 0.89, 0.81, and 0.91); quadratic mean diameter (R^2 = 0.80, 0.68, and 0.83); and density (R^2 = 0.74, 0.71, and 0.73) were predicted from low density (i.e., 0.5 point m^{-2}) LiDAR data for these 3 strata, respectively.

Key words: light detection and ranging, LiDAR, airborne laser scanning, forest modelling, remote sensing, forest stand variables, Great Lakes – St. Lawrence forest

RÉSUMÉ

Des modèles ont été élaborés afin d'effectuer des prédictions sur les variables des peuplements forestiers associées aux espèces communes des forêts des Grands Lacs et du Saint-Laurent dans le centre de l'Ontario au Canada, à partir des données de détection de la lumière et de calcul de la distance (LiDAR). Les peuplements ayant subi des coupes partielles de diverses intensités ou des traitements préliminaires d'espacement et établis sur diverses stations géographiques ont été retenus pour fins d'étude. Une stratification forestière générale a été mise en place et faisait référence (i) aux peuplements feuillus naturels, (ii) aux peuplements résineux naturels et, (iii) aux plantations de résineux. Les hauteurs maximales des peuplements (R^2 = 0,96, 0,98 et 0,98); la hauteur moyenne (R^2 = 0,86, 0,76 et 0,98); la surface terrière (R^2 = 0,80, 0,80 et 0,85); le volume (R^2 = 0,89, 0,81 et 0,91); le diamètre de la tige de surface terrière moyenne (R^2 = 0,80, 0,68 et 0,83) et la densité (R^2 = 0,74, 0,71 et 0,73) ont été prédites respectivement à partir de données LiDAR à faible densité (c'est-à-dire, 0,5 point m^{-2}) pour ces trois strates.

Mots clés : détection de la lumière et calcul de la distance, LiDAR, numérisation par laser aéroporté, modélisation forestière, télédétection, variables de peuplement forestier, forêt des Grands Lacs et du Saint-Laurent.

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Introduction

Evolving strategic- and operational-scale modelling activities requires the development of natural resource inventories with an increased spatial resolution of forest stand metrics and detailed terrain attributes. Forest modelling trends are shifting towards spatially explicit tools requiring increased accuracy of the location, accessibility, quantity (e.g., stand volume), and size (e.g., average piece size) to help formulate potential scenarios to be considered in developing forest management plans. Accurate estimates of inventory variables at the stand level become much more crucial when evaluating short- and long-term management decisions and alternatives with the use of spatially explicit models.

Current forest inventories are derived through airphoto interpretation and photogrammetric techniques that determine forested polygons and assign a species list, average height, site occupancy measure (i.e., stocking or crown closure) and a coarse estimate of age. With an interpreted height and estimated age, stand site productivity is derived (i.e., site class or site index). In Ontario, normal and empirical yield table estimates for an aggregated forest community are used to assign an average basal area and volume to a group of similar stand conditions based on an assessment of site productivity and species composition. Today, forest management modelling and operational planning require more accurate and precise estimates of these inventory variables.

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Airborne light detection and ranging (LiDAR) for forest applications has been studied since 1982 (Arp *et al.* 1982). Over the past decade, improvements in global positioning systems (GPS), inertial navigation systems (INS), computer hardware, LiDAR processing software, and reduced acquisition costs have permitted LiDAR technology to evolve from a research tool to operational status in forestry (Næsset 2004a, Stephens *et al.* 2007). Current acquisition costs, including classification of LiDAR points, breakline collection, derivation of digital elevation models (DEMs) and digital surface models (DSMs), has become an operationally affordable alternative when development of a precision inventory is considered.

Recent work in Ontario has focused on estimating forest inventory and biophysical variables for tolerant northern hardwoods (Lim *et al.* 2001, 2002, 2003; Todd *et al.* 2003, Lim and Treitz 2004), boreal mixedwoods (Thomas *et al.* 2006, 2008) and conifer plantations (Chasmer *et al.* 2006). Other international studies have also clearly demonstrated the application of LiDAR to forest inventory (Nilsson 1996; Lefsky *et al.* 1999a, b; Means *et al.* 1999; Holmgren 2003; Gobakken and Næsset 2004; 2007; Hyyppä *et al.* 2004; Næsset 2004b; Bollandsås and Næsset 2007; Breidenbach *et al.* 2007; Stephens *et al.* 2007).

The objective of this study was to explore the utility of airborne LiDAR data to estimate stand-level forest variables, which included stand top height (TOPHT), average height (AVGHT), stem density (DENSITY), quadratic mean diameter (QMDBH), basal area (SUMBA), and gross total volume (SUMGTV), for Great Lakes – St. Lawrence forest species with the goal of implementing these methods in operational inventories. This study is unique in comparison to previous studies in that it considers stands that have undergone various ranges of partial harvesting, which comprise much of the forest management activities for the Great Lakes – St. Lawrence forest, or initial spacing treatments, and are distributed across multiple geographic sites.

Material and Methods

Study area

The study area consisted of multiple research forest sites throughout central Ontario with the 2 primary sites consisting of the Swan Lake and Petawawa Research Forests (Fig. 1). Additional research plots, which were located in the Nipissing Forest, are representative of other Great Lakes – St. Lawrence forest types and silvicultural systems of central Ontario (Fig. 1).

Swan Lake Research Forest (SLRF)

The Swan Lake Research Forest is located 250 km north of Toronto and east of Huntsville, Ontario, within Algonquin Provincial Park. The 2000-ha site situated in Peck Township is located at 45° 28′ N, 78° 45′ W and ranges in elevation from 412 m to 587 m above sea level (asl). This site lies on the Precambrian Shield and is characterized by rolling hills and high rocky ridges that are separated by valleys scoured by glaciation. Outwash flats, ablation moraines, and drumlinoid deposits provide soil deposits ranging from coarse to medium texture. The Algonquin Dome, due to its elevation, has a climate that is generally cooler and wetter than its surrounding areas (Cole and Mallory 2005).

The site is within the Great Lakes – St. Lawrence Forest Region and comprises mature stands of shade- and mid-tolerant hardwoods (sugar maple [*Acer saccharum* Marsh.], American beech [*Fagus grandifolia* Ehrh.], soft maple [*Acer rubrum* L.], yellow birch [*Betula alleghaniensis* Britt.], ironwood [*Ostrya virginiana* (Mill.) K. Koch]), conifers (eastern hemlock [*Tsuga canadensis* (L.) Carrière], white pine [*Pinus strobus* L.], white spruce [*Picea glauca* (Moench) Voss], red spruce [*Picea rubens* Sarg.], eastern larch [*Larix laricina* (Du Roi) K. Koch], eastern white cedar [Thuja occidentalis L.], balsam fir [*Abies balsamea* (L.) Mill.]), and minor proportions of mid-tolerant and intolerant hardwoods (i.e., white birch [*Betula papyrifera* Marsh.], black cherry [*Prunus serotina* Ehrh.], white ash [*Fraxinus americana* L.], black ash [Fraxinus nigra Marsh.], and trembling aspen [*Populus tremuloides* Michx.]).

Petawawa Research Forest (PRF)

The Petawawa Research Forest (PRF) is located approximately 200 km west of Ottawa and 180 km east of North Bay, just east of Chalk River, Ontario (46° 00′ N, 77° 26′ W). The research forest encompasses 10 000 ha of mixed mature natural and plantation forest that is representative of the Great Lakes – St. Lawrence Forest Region and is characterized by eastern white pine, red pine, trembling aspen, and white birch. Red oak (*Quercus rubra* L.) dominates many poor, dry soils. Boreal forest species from the north and shade-tolerant hardwoods from the south exist on suitable sites.

PRF lies on the southern edge of the Precambrian Shield with its topography strongly influenced by glaciation and post-glacial outwashing. There are 3 types of terrains characterizing the site including extensive sand plains of mostly deltaic origin; imposing hills, shallow, sandy soils, and bedrock outcrops; and gently rolling hills with moderately deep, loamy sand containing numerous boulders. Elevations range from 140 to greater than 280 m asl within the forest. Mean annual precipitation for the research forest is around 82 cm per year. Approximately 25% of the precipitation falls as snow. The mean annual temperature is 4.4°C. The area averages 136 growing season days with an average of 100 days being frost-free.

Nipissing Forest (NF)

Nipissing Forest is an actively managed forest in Central Ontario. The Crown-managed forested area of 621 000 ha is centered on North Bay, Ontario (46° 18′ N, 79°27′ W). The forest is characterized as one of transition between the Great Lakes – St. Lawrence Forest Region and the Boreal Forest Region to the north. The main forest species include white pine, red pine, sugar maple, soft maple, red oak, yellow birch, trembling aspen and balsam fir.

Field data

Ground reference data were collected for the 3 study sites during the period of July 2005 and August 2006. Aside from establishing new plots for measurements, existing research plots were also used, which resulted in a network of plots that varied in shape (i.e., circular, square, and rectangular) and area (Table 1). The minimum plot area was 400 m^2. Conversely, there were a few large plots that ranged from 0.1 to 0.5 ha.

Each plot had all trees larger than or equal to 10 cm measured for diameter at breast height (DBH) with a diameter tape. Each tree was assessed for species, status (live or dead), crown class (dominant, co-dominant, etc.) and visual quality.

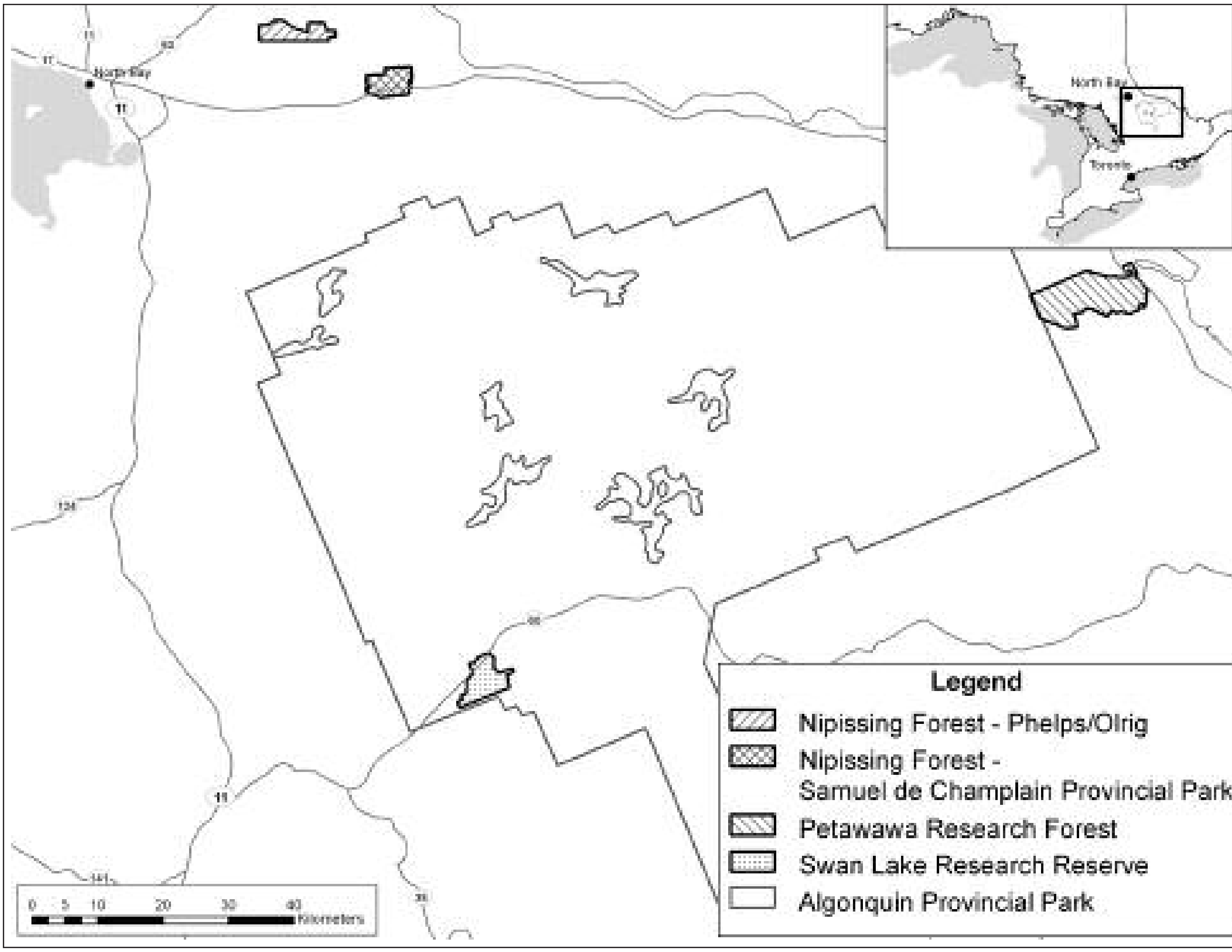

Fig. 1. Project study areas in Ontario.

Table 1. Field plots by research site and forest type

Research Site	Plot area (m^2)	Natural hardwoods	Natural conifers	Plantation conifers	Total
Swan Lake Research Forest	400	12	4	1	17
	800	6	–	–	6
	1000	4	–	–	4
Subtotal		**22**	**4**	**1**	**27**
Petawawa Research Forest	400	9	15	18	42
	506	–	–	1	1
	675	–	–	1	1
	1000	–	1	4	5
	1012	–	–	2	2
	1350	–	–	1	1
	2500	–	8	–	6
Subtotal		**9**	**24**	**27**	**60**
Nipissing Forest	400	6	3	–	–
	1000	9	–	–	–
Subtotal		**15**	**3**	**–**	**18**
Total		**46**	**31**	**28**	**105**

A Vertex™ hypsometer was used to measure height to base of live crown and total tree height for every tree. Heights of hardwood species were measured in the leaf-off period during fall 2005 and spring 2006.

The centre of each circular plot or corner post of each square or rectangular plot was geo-referenced with a Trimble Pro XT™ kinematic GPS unit connected to a Hurricane™ antenna, which was mounted on a tripod (Fig. 2). A minimum of 300 points was collected for each post position and later post-processed against a base station to achieve sub-meter accuracy. In cases where not all corner posts could be geo-referenced with GPS due to dense overhead canopy limiting the incoming signal, a compass was used to sight between plot posts to measure the bearing of the plot line and a fibreglass tape used to measure plot size.

The 3 research forests chosen for this study offered different forest ecosystems and silvicultural systems from which to sample and predict forest variables. An effort was made to sample extreme differences in density or basal area conditions in order to test the capability of LiDAR to penetrate the associated canopy conditions and its potential to model these ranges of density and basal area conditions.

SLRF offered tolerant hardwood forest conditions from no-harvest (i.e., natural old-growth areas) to stands that had undergone silvicultural treatment using single-tree selection methods. Additional conifer plots were also established within the SLRF.

The PRF offered plantation, natural unharvested, and silviculturally treated conifer stands. Plantations of different species and initial planting densities, along with natural stand controls and white and red pine stands treated with the uniform shelterwood system were sampled.

Three different site conditions were sampled within NF including: (i) the Olrig site consisting of young yellow birch stands exhibiting a range of residual basal area conditions; (ii) the Phelps site consisting of an unharvested red oak condition and a red oak uniform shelterwood treatment area; and (iii) natural white pine conditions at Samuel de Champlain Provincial Park.

The plot data were pooled for the 3 research sites, stratified, and analyzed according to 3 different forest species groups that included: (i) natural hardwoods, (ii) natural conifers; and (iii) plantation conifers. These coarse groupings were based primarily on species crown similarity (i.e., random oval, random conical, and orderly conical) with the assumption that LiDAR pulses would intercept each group in a unique way. Species statistics for these groupings are presented in Table 2. Plot data consisting of individual tree data were compiled to per hectare (ha) values using equations presented in Table 3.

Fig. 2. Acquiring sub-metre plot centre position.

Table 2. Field data statistics by forest species group

Natural hardwoods

		Top height (m)				Density (stems/ha)				Basal area (m^2/ha)				Gross total volume (m^3/ha)			
Leading species	**N**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**
Sugar Maple	27	24.1	16.8	27.3	2.9	389.2	200.0	630.0	103.0	22.3	11.6	36.1	6.9	201.5	76.6	341.1	72.4
Yellow Birch	9	18.9	15.8	25.2	3.2	417.8	340.0	530.0	69.4	11.1	6.1	18.8	4.5	84.0	40.1	166.2	46.4
Red Oak	3	24.7	23.4	25.7	1.2	275.0	75.0	600.0	284.0	16.8	5.5	35.6	16.4	155.3	52.7	319.7	143.9
Trembling Aspen	5	25.1	20.7	28.3	3.6	920.0	525.0	1475.0	359.0	24.4	17.6	28.8	4.4	234.2	144.9	274.1	52.7
American Beech	1	25.9	–	–	–	425.0	–	–	–	23.1	–	–	–	218.0	–	–	–
White Birch	1	23.0	–	–	–	525.0	–	–	–	25.4	–	–	–	210.7	–	–	–
All	**46**	**23.2**	**15.8**	**28.3**	**3.5**	**448.8**	**75.0**	**1475.0**	**226.2**	**20.1**	**5.5**	**36.1**	**8.2**	**179.6**	**40.1**	**341.1**	**84.4**

Natural conifers

		Top height (m)				Density (stems/ha)				Basal area (m^2/ha)				Gross total volume (m^3/ha)			
Leading species	**N**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**
Red Pine	11	28.3	22.4	33.2	3.4	325.0	68.0	775.0	245.1	32.5	13.1	60.3	14.8	368.3	165.3	724.1	172.7
White Pine	9	28.5	21.4	34.1	3.8	442.1	20.0	850.0	291.7	30.6	5.0	48.3	14.6	334.1	66.1	625.2	173.0
E. Hemlock	4	25.2	21.4	28.3	2.9	431.3	75.0	750.0	294.7	33.3	20.2	51.4	13.3	303.7	172.4	498.8	147.0
Black Spruce	2	22.5	22.1	23.0	0.6	787.5	500.0	1075.0	406.6	25.5	19.6	31.4	8.4	208.0	158.6	257.5	69.9
Balsam Fir	2	16.3	13.3	19.3	4.2	1102.5	730.0	1475.0	526.8	20.5	16.3	24.7	5.9	125.1	117.7	132.5	10.5
E. White Cedar	2	22.6	22.0	23.2	0.8	662.5	550.0	775.0	159.1	39.5	26.2	52.8	18.8	301.4	191.7	411.0	155.1
Jack Pine	1	24.2	–	–	–	350.0	–	–	–	19.9	–	–	–	194.0	–	–	–
All	**31**	**26.3**	**13.3**	**34.1**	**4.5**	**474.5**	**20.0**	**1475.0**	**333.8**	**30.9**	**5.0**	**60.3**	**13.5**	**314.1**	**66.1**	**724.1**	**160.9**

Conifer plantations

		Top height (m)				Density (stems/ha)				Basal area (m^2/ha)				Gross total volume (m^3/ha)			
Leading species	**N**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**	**Mean**	**Min**	**Max**	**Std Dev**
Larch	2	25.8	22.8	28.8	4.2	507.5	340.0	675.0	236.9	29.5	28.5	30.6	1.5	275.3	240.2	310.4	49.7
Jack Pine	5	15.0	10.3	21.7	5.4	739.8	320.0	1200.0	425.8	16.0	6.6	25.1	8.2	109.7	34.5	212.6	84.8
Red Pine	3	28.2	27.7	28.9	0.6	1108.7	742.0	1502.0	380.7	67.9	66.6	69.9	1.8	786.9	764.4	803.0	20.1
Black Spruce	4	14.6	12.7	16.4	2.0	1481.3	1050.0	1900.0	350.8	25.9	16.1	30.7	6.6	149.4	76.7	193.3	52.6
Red Spruce	2	16.6	15.4	17.8	1.7	1137.5	1125.0	1150.0	17.7	37.0	26.2	47.8	15.2	264.3	168.9	359.8	135.0
White Spruce	12	17.9	10.8	25.2	3.2	1075.0	340.0	2700.0	648.6	30.0	17.1	44.2	8.6	198.3	87.6	292.5	68.7
All	**28**	**18.5**	**10.3**	**28.9**	**5.4**	**1040.7**	**320.0**	**2700.0**	**541.4**	**31.4**	**6.6**	**69.9**	**15.9**	**248.8**	**34.5**	**803.0**	**206.2**

Table 3. Ground metrics calculated from field data

Ground metric	Description
TOPHT (m)	Calculated as the average of the largest 100 stems per hectare
AVGHT (m)	Calculated as the average height of all trees 10.0cm and larger.
DENSITY (stems ha^{-1})	Number of live trees 10.0cm and larger expressed per hectare
QMDBH (cm)	$\sqrt{\left[\left(\sum DBH^{2}/n\right)\right]}$ where n is stems per plot
SUMBA (m^2 ha^{-1})	DBH^2 * .00007854 Per hectare value calculated by summing each tree per plot.
SUMGTV (m^3 ha^{-1}) (Honer *et al.* 1983)	$= ß_1 * DBH^{2}*(1\text{-}0.04365* ß_2)^2/(ß_3+(0.3048 * ß_4/Ht))$ Individual tree volume equation. Coefficients varies by species Per hectare value calculated by summing each tree per plot.

LiDAR data

LiDAR data were collected in September 2005 using an upgraded Leica ALS40 airborne laser scanner mounted in a King Air 90 aircraft. The base mission was flown at 9000 feet with a 20° field of view, scan rate of 30 Hz, and a maximum pulse repetition frequency of 32 300 Hz. This configuration resulted in a cross track spacing of 2.87 m, an along-track spacing of 2.4 m, an average sampling density of 0.46 points m^{-2}, and a swath width of approximately 1 km. The LiDAR point cloud data were classified as ground or vegetation by the vendor using proprietary algorithms. LiDAR returns that were classified as ground were normalized to the terrain using a Triangulated Irregular Network (TIN). For each return classified as vegetation, the z-value on the TIN matching its x–y coordinate was subtracted from the return's z-value resulting in a normalized height measure (metres above ground). These normalized data were in turn used to generate various LiDAR-based statistical, canopy height, and canopy density predictors

LiDAR-based predictors

Three types of predictors, which included statistical, canopy height, and canopy density predictors, were derived from all LiDAR returns (i.e., classified ground and vegetation returns). No height threshold was used to filter any of the point data. The statistical group of predictors included mean height and standard deviation, absolute deviation, skew, and kurtosis of the distribution of LiDAR height measurements. The canopy height predictors consisted of deciles of LiDAR canopy height (i.e., p1 … p9; with p1 and p9 corresponding to the 1^{st} and 9^{th} deciles, respectively) and the maximum LiDAR height. A decile is any of the 9 values that divide sorted data into 10 equal parts with each part representing 1/10th of the sample or population. For example, the 5^{th} decile, also referred to as the 50^{th} percentile, 2^{nd} quartile, or median, of LiDAR canopy height would correspond to the height value where 50% of the observations are found below and above it.

Eleven canopy density metrics were derived. For each plot, the range of LiDAR height measurements was divided into 10 equal intervals and the cumulative proportion of LiDAR returns found in each interval, starting from the lowest interval (i.e., d1), was calculated. Since the last interval always sum to a cumulative probability of 1, it was excluded resulting in 9 canopy density metrics (i.e., d1 … d9). The remaining 2 canopy density metrics were calculated as the proportion of first returns divided by all returns intersecting a sample plot (Da) and the proportion of first returns classified as vegetation (i.e., non-ground) divided by all returns intersecting a sample plot (Db).

Statistical analyses

Best subsets regression, a model-building technique that identifies subsets of variables that best predict responses on a dependent variable by linear or non-linear regression, was used in this study. For each forest variable, the 4 best models were identified for models that were based on 1 up to 10 predictors. For example, for a model based on 5 predictors, the 4 "best" predictors were identified and assessed.

A diagnosis of each model was performed to determine if parametric statistical assumptions were satisfied. The Shapiro–Wilk's *W* Test was used to determine if residuals were normally distributed, whereas the Modified Levene's Test was used to check for the presence of heteroschedasticity (i.e., unequal error variance). In many instances, dependent variables were transformed using a natural log transformation in order to satisfy these assumptions. Back transformations were based on the method described by Baskerville (1972) where ½ of the squared residual standard error is added to the estimate before taking the inverse log.

As LiDAR predictors have been reported to be highly correlated, the variance inflation factors (VIF) for the predictors used in a model were examined (Neter *et al.* 1996). Candidate models where predictors exhibited VIF greater than 10 were discarded, as values above 10 suggest the presence of multicollinearity in the predictor data (Neter *et al.* 1996). The final models (Table 4), in addition to the individual predictors used in each model, were tested at the 0.05 significance level.

The cross-validation prediction of sum of squares (PRESS) procedure (Myers 1986) was used to validate the models, since not enough observations were available to permit the splitting

Table 4. Model results for the natural hardwoods, natural conifers and plantation conifers forest grouping

Variable	Equation	R^2	R^2(adj)	*p*	RMSE (%)	Shapiro–Wilk's Test		Modified Levene's Test		PRESS RMSE (%)
						W	*p*	t_L	*p*	
Natural hardwoods										
SUMBA (m^2 ha^{-1})	SUMBA = 69.0 - 3.88 abs_dev - 65.0 Da + 1.51 p70 + 29.7 d6 - 25.2 d8	0.82	0.80	< 0.001	3.46 (17.2)	0.98	0.74	0.12	0.90	3.99 (19.9)
SUMGTV (m^3 ha^{-1})	ln(SUMGTV) = 6.56 - 0.156 abs_dev - 3.44 Da + 0.128 p70 + 1.85 d7 - 1.90 d8	0.90	0.89	< 0.001	39.35 (21.9)	0.97	0.25	1.54	0.13	52.03 (29.0)
DENSITY (stems ha^{-1})	ln(DENSITY) = 4.22 - 3.65 Da + 5.57 Db - 0.0363 p70 + 3.04 d1 - 2.27 d4	0.77	0.74	< 0.001	196.03 (43.7)	0.97	0.38	1.81	0.08	214.98 (47.9)
QMDBH (cm)	ln(QMDBH) = 2.93 - 0.141 abs_dev - 1.35 Db + 0.101 p70 + 1.58 d5	0.82	0.80	< 0.001	3.07 (12.4)	0.97	0.29	0.02	0.99	4.17 (16.8)
AVGHT (m)	AVGHT = 2.41 + 1.18 std_dev - 1.91 skew - 0.363 p60 + 0.738 p70	0.87	0.86	< 0.001	1.10 (5.7)	0.96	0.12	1.60	0.12	1.25 (6.4)
TOPHT (m)	ln(TOPHT) = 2.13 + 0.0242 p70 + 0.0225 max + 0.191 d4	0.96	0.96	< 0.001	0.80 (3.5)	0.97	0.22	0.11	0.92	0.89 (3.8)
Natural conifers										
SUMBA (m^2 ha^{-1})	ln(SUMBA) = 2.17 + 0.0576 p50 + 0.0859 p80 - 0.0887 max + 1.22 d8	0.82	0.80	< 0.001	7.23 (23.4)	0.97	0.54	0.74	0.47	11.46 (37.1)
SUMGTV (m^3 ha^{-1})	ln(SUMGTV) = 2.63 + 0.175 abs_dev - 0.466 skew + 1.74 Db	0.83	0.81	< 0.001	72.96 (23.2)	0.97	0.64	0.73	0.47	106.84 (34.0)
DENSITY (stems ha^{-1})	ln(DENSITY) = 8.54 + 0.250 abs_dev + 0.0783 p50 - 0.211 max	0.74	0.71	< 0.001	222.27 (46.8)	0.99	0.94	0.17	0.87	611.10 (128.6)
QMDBH (cm)	ln(QMDBH) = 2.27 + 0.207 mean + 0.287 skew - 0.0919 p50	0.71	0.68	< 0.001	6.93 (20.1)	0.98	0.88	0.09	0.93	10.51 (30.5)
AVGHT (m)	AVGHT = 13.4 - 0.546 p50 + 1.01 max - 15.6 d7	0.78	0.76	< 0.001	2.54 (11.5)	0.96	0.24	0.70	0.49	2.85 (12.9)
TOPHT (m)	TOPHT = 18.8 + 0.104 p50 + 0.751 max - 14.5 d9	0.96	0.96	< 0.001	0.89 (3.4)	0.94	0.11	0.95	0.35	0.99 (3.8)
Plantation conifers										
SUMBA (m^2 ha^{-1})	ln(SUMBA) = 1.95 + 0.115 p40 - 1.81 d4 + 1.42 d7	0.87	0.85	< 0.001	5.33 (17.0)	0.97	0.65	0.49	0.63	7.46 (23.7)
SUMGTV (m^3 ha^{-1})	ln(SUMGTV) = 2.28 - 0.467 skew + 0.156 p50 + 1.66 d7	0.92	0.91	< 0.001	40.18 (16.2)	0.99	0.96	0.20	0.84	59.69 (24.0)
DENSITY (stems ha^{-1})	ln(DENSITY) = 6.82 + 0.484 std_dev + 0.173 p30 - 0.214 max	0.76	0.73	< 0.001	257.08 (24.7)	0.98	0.79	0.82	0.42	486.99 (46.8)
QMDBH (cm)	ln(QMDBH) = 0.259 + 1.23 Da - 0.0336 p30 + 0.0600 max + 1.02 d9	0.86	0.83	< 0.001	2.34 (11.4)	0.93	0.07	0.74	0.47	3.17 (15.4)
AVGHT (m)	AVGHT = 3.97 + 0.388 kurtosis - 0.294 p30 + 0.950 p90	0.98	0.98	< 0.001	0.59 (3.7)	0.96	0.36	0.12	0.90	0.71 (4.5)
TOPHT (m)	TOPHT = 4.27 + 0.977 p90	0.98	0.98	< 0.001	0..65 (3.5)	0.97	0.60	1.03	0.31	0.75 (4.1)

of the data into separate model development and validation datasets. This approach to model validation is similar to applying the equation to an independent sample because the PRESS residual is obtained for the observations that are not included in the data when the equation is derived (Sun *et al.* 2003).

The procedure for calculation of the PRESS statistic is undertaken by: (1) fitting a regression model through the data minus 1 observation, 2) obtaining the predicted value of the excluded observation, 3) calculating the residual for the predicted value (observed–predicted), 4) repeating steps 1 to 3 for the remaining observations, 5) calculating the sum of squares of all residuals, and 6) deriving the PRESS statistic, herein referred to as the PRESS RMSE, by calculating the square root of the sum of squares of the residuals divided by the total number of observations (Myers 1986).

Results and Discussion

Results indicate that LiDAR statistically based, canopy height, and density predictors are suitable predictors of stand level variables for enhanced inventory development for forest groupings within the Great Lakes – St. Lawrence forest (Fig. 3–8 and Table 4). In addition, it appears that adequate models can be developed from plot data covering a range of silvicultural treatments and spanning large geographies even when low sampling density LiDAR data (i.e., 0.5 hits m^{-2}) are used.

An evaluation of low-density to high-density LiDAR data for the prediction of boreal mixedwood stand attributes was reported by Thomas *et al.* (2006). Results of the validation efforts concluded that low- and high-density LiDAR models were highly correlated with key forest structural attributes, mean dominant height, basal area, and average aboveground biomass (low density: $R^2 = 0.90$, 0.91, and 0.92; and high density: $R^2 = 0.84$, 0.89, and 0.91). Næsset (2004c) also found that LiDAR density, as a function of flying height, had no statistical significance on the prediction of biophysical stand characteristics derived from regression equations.

The broad species models developed for this study identify the potential for further refinement of model predictions based on more rigorous stratification and field sampling. The results reported here are based on field data that were pooled from across many geographic sites into strata arranged by general species crown shapes. Field data were collected across large gradients of age, height, basal area, density, species, and silvicultural treatment to identify the capability of LiDAR to penetrate various crown closure levels and to determine the success of generalized forest variable predictive models. Operational application of LiDAR for forest inventory would involve more rigorous stratification of field sampling efforts within a single forest being investigated (Næsset 2004a). As a result, it is assumed that model predictions presented below could be further improved within an operational inventory context.

Top height and average height

Simple predictive models for TOPHT (i.e., the height of the largest 100 trees ha^{-1}) were developed for each of the 3 forest groupings. The TOPHT model for all groupings performed very well and as anticipated given that LiDAR pulses first intercept the top (or near the top) of crowns within a plot (Table 4; Fig. 3). The coefficients of determination (R^2) for natural hardwoods and natural conifers were 0.96, whereas a value of 0.98 was obtained for plantation conifers. The range of root mean square error (RMSE) values across all forest groupings ranged from 0.70 m to 0.89 m (3%–4%). The results reported here are consistent with those that have been reported in the literature (e.g., Lim *et al.* 2003, Holmgren and Jonsson 2004, Næsset 2004a, Rooker Jensen *et al.* 2006, Thomas *et al.* 2006).

The larger error in the prediction of TOPHT for the natural conifer grouping (i.e., RMSE = 0.89) is assumed to be due to the fact that TOPHT was calculated from field plot data in stands that had undergone a uniform shelterwood silvicultural treatment. TOPHT calculated from field data biased the expression of TOPHT by utilizing the largest trees that were left on site to be a residual seed source and as such, were spaced around the experimental block. In some cases, large trees were removed for the purpose of crown spacing, resulting in smaller trees being used to calculate an average TOPHT. In general, TOPHT was over-estimated on these sites.

The prediction of AVGHT was generally poorer than that for TOPHT (i.e., R^2 values for natural hardwoods, natural conifers, and plantation conifers were 0.87, 0.78, and 0.98, respectively) (Table 4; Fig. 4). In many respects, these results are expected given the variability in vertical structure characterizing the various forest groups considered. The exception is conifer plantations where the RMSE for the prediction of AVGHT is comparable to TOPHT (0.59 m and 0.65 m, respectively). Conifer plantations, differing from the other species groups investigated, have a simple single-tier vertical structure profile. The natural hardwood and natural conifer conditions sampled in this study have a multi-tier structure profile owing to their natural adaptive recruitment strategy (i.e., gap phase replacement in a tolerant hardwood stand developing a mixed-age and height cohort) or their human manipulated structure from silvicultural activities (i.e., single-tree selection or uniform shelterwood treatments). The relatively low LiDAR sampling density (Fig. 9) may also have contributed to the poorer performance of this model given that there are fewer ground samples in stands with mixed-species, mixed-height structures (Thomas *et al.* 2006).

Although larger than the RMSE values reported for TOPHT, the reported RMSE values for AVGHT predictions we are still deemed acceptable for stand inventory purposes. The reported RMSE of 1.10m (6%) for hardwoods provideds an indication of the range of vertical structure found normally in these multi-tiered unenven-aged stands (i.e., saplings through to sawlog-sized trees). Natural conifers, weighted heavily in the dataset by the influence of 10-year, post-treatment white pine shelterwood plots with a large amount of regeneration, reported a large RMSE of 2.54 m (12%). This result is not surprising as these types of stand conditions (refer to Fig. 9) consist of tall overstory trees and advanced regeneration that will intercept LiDAR pulses. Conifer plantations reported the lowest RMSE of 0.59 m (4%) as expected with their simpler canopy structure. Researchers from other jurisdictions have reported similar RMSE values for average height model predictions (Holmgren 2003, Holmgren and Jonsson 2004, Næsset 2004a).

Stocking (basal area and density)

The prediction of stand stocking attributes (i.e., density and basal area) based on LiDAR predictors has proven more dif-

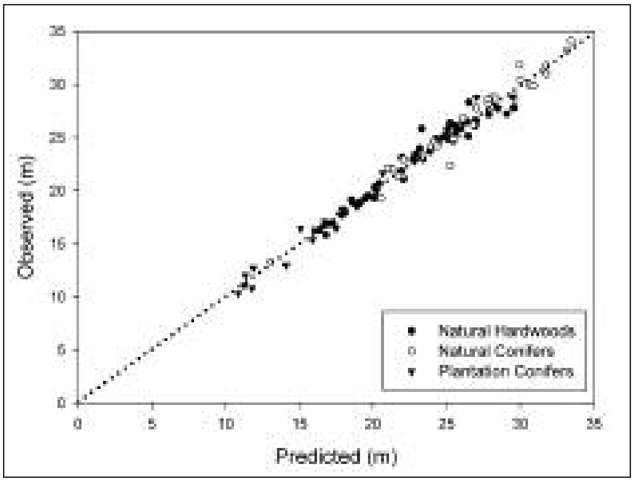

Fig. 3. Top height (TOPHT) field observed values vs. predicted model values for the natural hardwoods, natural conifers and plantation conifers forest groupings.

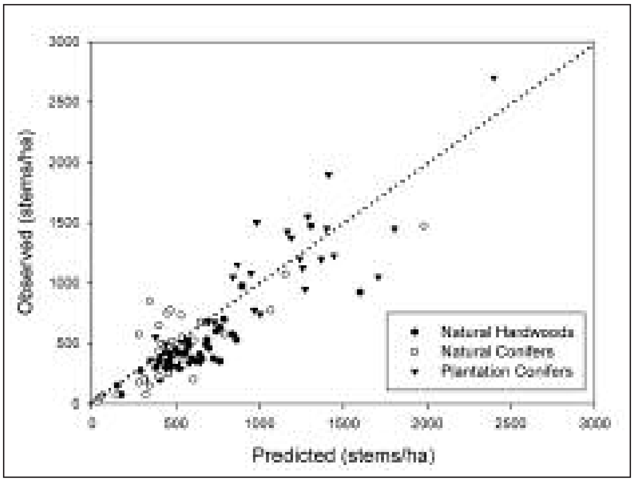

Fig. 6. Density (number of trees) field observed values vs. predicted model values for the natural hardwoods, natural conifers and plantation conifers forest groupings.

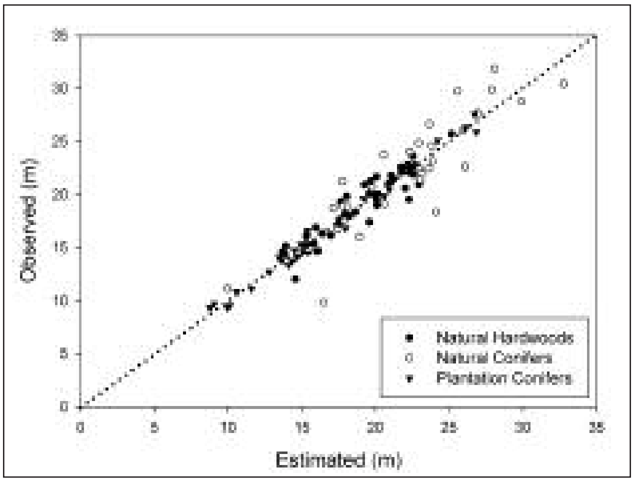

Fig. 4. Average height (AVGHT) field observed values vs. predicted model values for the natural hardwoods, natural conifers and plantation conifers forest groupings.

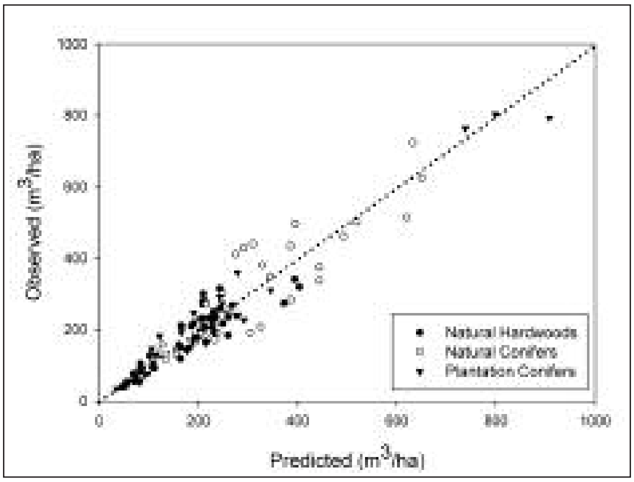

Fig. 7. Volume (SUMGTV) field observed values vs. predicted model values for the natural hardwoods, natural conifers and plantation conifers forest groupings.

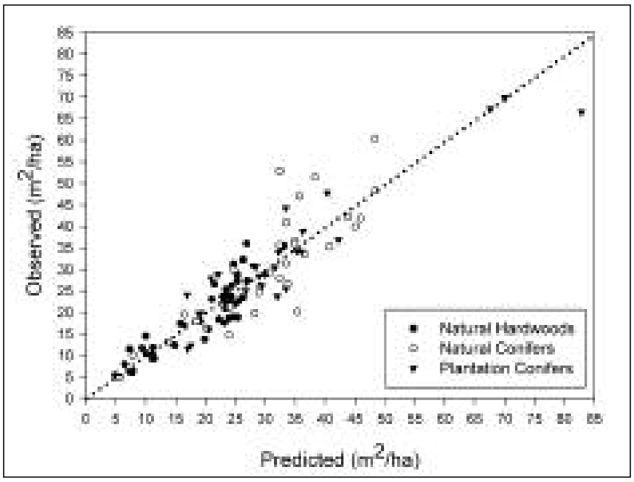

Fig. 5. Basal area (SUMBA) field observed values vs. predicted model values for the natural hardwoods, natural conifers and plantation conifers forest groupings.

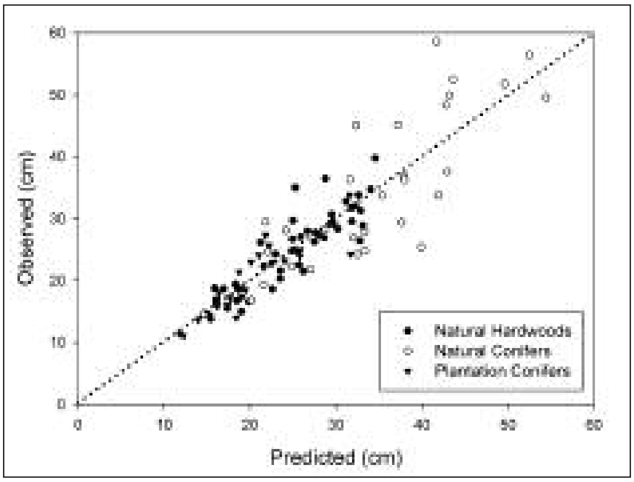

Fig. 8. Quadratic mean diameter (QMDBH) field observed values vs. predicted model values for the natural hardwoods, natural conifers and plantation conifers forest groupings.

Fig. 9. Comparison of raw LiDAR hits of different densities for various forest stand conditions. Green dots indicate classified vegetation hits. Purple dots indicate classified ground hits.

ficult than predicting height metrics (Stephens *et al.* 2007). Basal area models developed for this study exhibit R^2 values of 0.80, 0.79, and 0.85 with associated RMSE values of 3.46 m^2 ha^{-1} (17%); 7.23 m^2 ha^{-1} (23%); and 5.33 m^2 ha^{-1} (17%) for natural hardwoods, natural conifers, and conifer plantations, respectively (Table 4; Fig. 5). These values fall within the range of RMSE values reported by other authors, such as Drake *et al.* (2002) (7.15 – 7.88 m^2ha^{-1}), Holmgren (2003) (4.8 m^2 ha^{-1}), Holmgren and Jonsson (2004) (3.0 m^2 ha^{-1}), Næsset (2004a) (2.38 – 4.88 m^2 ha^{-1}), Rooker Jensen *et al.* (2006) (3.1 m^2 ha^{-1}), and Stephens *et al.* (2007) (8.0 m^2 ha^{-1}).

Many forest management activities are driven by knowledge of stand basal area. These include the selection of an appropriate silvicultural system in combination with a prescribed thinning regime development. The performance of the basal area model may be a function of the broad species groupings combined with the wide range of stand spacing and treatments sampled. The hardwood grouping includes tolerant hardwood conditions with a wide range of basal area conditions along with varied structures (i.e., managed to old growth). Many of the stands sampled were uneven-aged and consisted of a wide range of size classes. This grouping also included mid-tolerant hardwood plots (i.e., red oak and yellow birch) some of which had recently undergone spacing and uniform shelterwood treatments within 2 years of the LiDAR acquisition. Plots dominated by intolerant trembling aspen and white birch species were also included in this grouping. Although the crown shapes of these species are considered similar (i.e., random oval), measurable stand attributes, such as basal area, diameter distribution and stand density, can be very different for these tolerant, mid- and intolerant species groups. These intolerant stands tend to exhibit a normal distribution of stem sizes versus the inverse J-shaped curve of size classes commonly associated with tolerant species. As a result, variable potential for low- and mid-storey stems to be sampled by LiDAR pulses in the tolerant species plots may be causing different patterns than found in the intolerant-dominated species plots with no significant understory.

Whereas volume predictions have been successful in a number of studies, prediction of stem density has been shown to be more problematic (Maltamo *et al.* 2004). The reason for the discrepancy between volume and stem density prediction has been attributed to the fact that the majority of the volume is accounted for in the dominant tree layer (Vuokila 1980; from Maltamo *et al.* 2004), which accounts for the dominant component of the LiDAR data distribution. In this study, prediction of stem density from LiDAR predictors was the least successful application. RMSE values ranged from 196 stems ha^{-1} (44%) for hardwoods, 222 stems ha^{-1} (47%) for natural conifers, and 257 stems ha^{-1} (25%) for plantation conifers (Table 4; Fig. 6).

Maltamo *et al.* (2003) addressed the problem of estimating stem density by calibrating the estimates using theoretical diameter distributions to describe small trees. However, this modification for estimating stem density was applied to high-resolution optical and LiDAR data in a single-tree approach. Here, with the application of low-sampling-density LiDAR data, a large RMSE for the natural hardwoods group may be a result of the inclusion of trembling aspen plots. These plots exhibit densities uncommon for the tolerant and mid-tolerant species comprising the rest of the grouping, perhaps further biasing the predictive model. A potential reason for the natural conifer model results may be the excessive variation in species and/or vertical crown distribution. Many of the plots had been silviculturally treated and now have a dense understory of regeneration present in the plots. Their associated LiDAR canopy height and canopy density predictors along with unharvested plot conditions may have caused noise in the model development. It is anticipated that model performance could be improved given more dense LiDAR point data (i.e., higher sampling point density).

Volume

Volume estimation for forest stands is a critical piece of information required for strategic and operational modelling of wood supply. Predictions of gross total volume resulted in RMSE values of 39.4 m^3 ha^{-1} (22%), 73.0 m^3 ha^{-1} (23%), 40.2 m^3 ha^{-1} (16%) for natural hardwoods, natural conifers, and plantation conifers, respectively (Table 4; Fig. 7). The R^2 values for the natural hardwoods, natural conifers, and plantations conifers were 0.90, 0.83, and 0.92, respectively. These results are in agreement with those reported by others (Holmgren 2003 [55 m^3 ha^{-1}], Holmgren and Jonsson 2004 [28 m^3 ha^{-1}], Næsset 2004a [13.9 – 45.9 m^3 ha^{-1}], Rooker Jensen *et al.* 2006 [23.7 m^3 ha^{-1}]).

Quadratic mean diameter

In addition to basal area and volume, knowledge of a stands QMDBH (cm) provides key information to forest planners and operational forest managers as it relates directly to horizontal structure for habitat, expected product size distributions and associated harvesting efficiencies. Favourable results from this study (RMSE values of 3.1 cm (12%), 6.9 cm (20%), and 2.3 cm (11%) for natural hardwoods, natural conifers and plantation conifers, respectively) support LiDAR's predictive ability to enhance inventories with key attributes not normally found in current inventories (Table 4; Fig. 8). Results from other studies support the ability of LiDAR to predict QMDBH within this range (Drake *et al.* 2002 (RMSE = 3.74 cm to 3.84 cm), Holmgren and Jonsson 2004 (RMSE = 1.9 cm), Rooker Jensen *et al.* 2006 (RMSE = 9.3 cm)). As identified earlier in the discussion of the other predicted forest variables, a more refined stratification and modest increase in LiDAR density are expected to reduce the RMSE values presented in this study.

Validation

As expected, the model validation resulted in an increase in the resultant PRESS RMSE values when compared to the model RMSE values (Table 4). Not surprisingly, only minor differences were found through the validation of the height estimation model for all forest groupings. Generally, the natural hardwood model validation effort demonstrated reasonable stability in each of the derived models. The exception to this is the density model in the hardwoods and also in the other forest groupings. This result is not unexpected as this equation represented the poorest model fits of this study. Better estimates of density for all forest groupings may require refined stratification or increased LiDAR sampling densities. The natural conifer and the plantation conifer groupings validation results showed similar trends to those of the natural hardwood except that larger increases in RMSE generally were observed. These increases are a function of using broad forest groupings for model development and the extreme range of density conditions targeted for plot establishment.

Conclusion

In the past 5 years, LiDAR has moved from the research arena to operational reality in the area of forest resource inventory. LiDAR has proven an efficient and accurate means to spatially estimate stand height and forest structure (i.e., DENSITY, SUMBA, QMDBH and SUMVOL) in support of statistically based model predictions across a broad range of forest types. Decreased acquisition costs, increased sampling densities, and enhanced inventory variable estimation have now made this technology attractive to operational forest inventories across a range of conditions. This enhanced capacity is critical to decision makers for sustainable forest management.

The results of this study clearly support the contribution of discrete-return LiDAR data for the enhancement of forest inventories for species groups in the Great Lakes – St. Lawrence forest. It has been demonstrated that low sampling density LiDAR data (0.5 hits m^{-2}) provide acceptable levels of accuracy for forest level attributes such as TOPHT, AVGHT, SUMBA, SUMGTV, DENSITY, and QMDBH. In addition, the models and results were based on a stratification of broad forest crown shapes and field plots from multiple geographical locations. Future studies will focus on developing improved models for more precise forest groupings and identifying appropriate ranges of LiDAR sampling densities to best predict this suite of forest variables.

Acknowledgements

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