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# Estimation of Above ground Forest Biomass from Airborne Discrete Return Laser Scanner Data Using Canopy-based Quantile Estimators

KEVIN S. LIM and PAUL M. TREITZ

*Department of Geography, Faculty of Arts and Science, Queen's University, Kingston, Ontario, Canada, K7L 3N6*

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A conceptual model describing why laser height metrics derived from airborne discrete return laser scanner data are highly correlated with above ground biomass is proposed. Following from this conceptual model, the concept of canopy-based quantile estimators of above ground forest biomass is introduced and applied to an uneven-aged, mature to overmature, tolerant hardwood forest. Results from using the 0th, 25th, 50th, 75th and 100th percentiles of the distributions of laser canopy heights to estimate above ground biomass are reported. A comparison of the five models for each dependent variable group did not reveal any overt differences between models with respect to their predictive capabilities. The coefficient of determination ($r^2$) for each model is greater than 0.80 and any two models may differ at most by up to 9%. Differences in root-mean-square error (RMSE) between models for above ground total, stem wood, stem bark, live branch and foliage biomass were 8.1, 5.1, 2.9, 2.1 and 1.1 Mg ha$^{-1}$, respectively.
*Key words: Above ground forest biomass, airborne laser scanning, forest structure, laser altimetry, LIDAR, quantile estimators, remote sensing.*

Correspondence to: K. S. Lim, e-mail: 9KK14@qlink.queensu.ca

## INTRODUCTION

Light detection and ranging (LIDAR) is a remote sensing technology that can be used to estimate various forest biophysical properties (e.g. Ritchie et al. 1993, Nilsson 1996, Næsset 2002, Holmgren et al. 2003) and characterize forest canopy elements in three dimensions (e.g. Harding et al. 2001, Lovell et al. 2003), while accurately mapping the terrain below forest canopies (Reutebuch et al. 2003, Hodgson & Bresnahan 2004). Previous research related to LIDAR remote sensing of forest biophysical properties and canopy structure is reviewed by Lefsky et al. (2002) and Lim et al. (2003*b*). Airborne LIDAR instruments that are relevant for remote sensing of forests are all pulse ranging instruments that record multiple laser returns (i.e. discrete return) or digitize the entire amplitude of the backscattered energy over time (i.e. full waveform) from each laser pulse. Moreover, these instruments can differ from one another with respect to how they sample the surface of the Earth. Airborne LIDAR instruments that distribute laser postings along a path following the trajectory of the aircraft are referred to as laser profiling instruments, whereas laser scanning instruments distribute laser postings in the along- and across-track directions. Above ground forest biomass has been estimated using: (1) full waveform laser scanning (Means et al. 1999, Lefsky et al. 1999, Drake et al. 2002*a*, 2002b, 2003); (2) discrete return laser profiling (Nelson et al. 1988*a*, 1988b, 1997, 2004); and (3) discrete return laser scanning (Lim et al. 2003*a*).

Irrespective of the type of LIDAR remote sensing instrument used, the general approach of all previous studies has been to use some physical measurement of forest canopies (e.g. mean canopy height) as derived from the laser data to estimate above ground biomass. Although the majority of studies have been, in general, successful in estimating above ground biomass, commonalities between reported predictor variables used in model development are rare. Consequently, those predictor variables reported in previous studies are likely to be study- and site-specific, and unable to be generalized to other forest types and/or site conditions. By examining the nature of the empirical relationships found in previous studies so as to understand why metrics derived from laser data can be used to estimate above ground biomass, the potential exists to develop new alternative novel estimators of above ground biomass for application to a range of forest types and stand conditions.

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Following a review of previous studies of above ground biomass estimation using different types of LIDAR instruments, a conceptual model describing why laser height metrics derived from airborne discrete return laser scanner data are highly correlated with above ground biomass at the plot and stand levels is proposed. This conceptual model stems from research by Magnussen & Boudewyn (1998) and on well-known allometric relationships between individual components of biomass. Following from the conceptual model, the concept of canopy-based quantile estimators is introduced. It is hypothesized that these types of predictor variable that are derived from discrete return laser data are correlated with above ground biomass and are applicable for a range of forest types. The results of the application of canopy-based quantile estimators in a study to estimate above ground total, stem wood, stem bark, live branch and foliage biomass for a tolerant hardwood forest are reported. Hence, the focus of this paper is on evaluating the potential of various canopy-based quantile estimators for modelling above ground biomass. This represents a first step towards exploring the applicability of canopy-based quantile estimators for deciduous forest environments and identifying possible limitations associated with canopy-based quantile estimators of above ground biomass.

### *Previous research*

Using laser data acquired with the Scanning Lidar Imager of Canopies by Echo Recovery (SLICER; Blair et al. 1994), Means et al. (1999) and Lefsky et al. (1999), using similar data analysis and processing techniques, demonstrated the capabilities of SLICER to estimate above ground biomass for Douglas fir [*Pseudotsuga menziesii* (Mirbel) Franco] and western hemlock [*Tsuga heterophylla* (Raf.) Sarg.] in the western Cascade Range, Oregon, USA, and for deciduous forests in eastern Maryland, USA. Key predictor variables used in these studies generally consisted of laser height metrics derived from canopy height profiles or the sum of the portion of waveform return reflected from the canopy or ground. Means et al. (1999) reported that models using: (1) mean canopy height (LHt); (2) quadratic mean canopy height (QMCH) and LHt; and (3) the sum of the portion of waveform return from the canopy, QMCH and LHt as predictor variables accounted for 90% [root-mean-square error (RMSE) = 132 Mg ha$^{-1}$], 94% (RMSE = 103 Mg ha$^{-1}$) and 96% (RMSE = 88 Mg ha$^{-1}$) of variation in the above ground biomass observations, respectively. Furthermore, foliage biomass was estimated using the sum of the portion of waveform return from the canopy or ground with the coefficient of determination ($r^2$) ranging from 0.67 to 0.84 (RMSE = 1.3–2.0 Mg ha$^{-1}$). Lefsky et al. (1999) reported similar results with respect to the predictive capabilities of laser height metrics derived from canopy height profiles. Simple linear models using maximum canopy height, median canopy height, mean canopy height and QMCH were reported to account for 80%, 70%, 73% and 80% of variation in the above ground biomass observations, respectively.

Above ground biomass estimation studies using the Laser Vegetation Imaging Sensor (LVIS; Blair et al. 1999) have been reported by Drake et al. (2002*a*, 2002*b*, 2003). These studies have primarily focused on the tropical forests of Costa Rica and Panama. Drake et al. (2002*a*) explored four metrics derived from the waveforms for above ground biomass estimation: (1) LIDAR canopy height; (2) the height of median energy, referred to as the HOME metric; (3) a height/median ratio (i.e. HOME divided by LIDAR canopy height); and (4) a ground return ratio based on the proportion of total intensity found in the last Gaussian peak of the waveform. The model using the HOME metric as a single predictor variable was deemed to be the best overall model as it accounted for 89% of variance in above ground biomass and produced a cross-validated RMSE of 22.54 Mg ha$^{-1}$. Moreover, the relationship found between above ground biomass and the laser height metric was non-asymptotic. Unlike the other laser height metrics explored, the predictive capabilities of the HOME metric were attributed to its sensitivity to the vertical organization and density of canopy structural elements (Drake et al. 2002*a*). The subsequent studies by Drake et al. (2002*b*, 2003) have not focused on above ground biomass estimation, but rather on exploring the relationship between vertical canopy profiles derived from field data and laser-based canopy height profiles. Here, the authors address the reasons as to why laser height metrics are correlated with above ground biomass (Drake et al. 2002*b*), and whether the relationship between laser height metrics (i.e. HOME) and above ground biomass could be generalized to other tropical forest regions and types (Drake et al. 2003).

Nelson et al. (1988*b*) estimated above ground biomass along transects for pine plantations of varying age and canopy densities in south-western Georgia, USA, using a discrete return airborne laser profiling instrument. Six laser height metrics were individually explored as predictors of above ground biomass, where each laser height metric differed from one another with respect to the proportion of laser returns considered within a sample plot. The authors found that the majority of models evaluated were all similar to one another with respect to their predictive capabilities and that the similarities between models were due to the high correlation between the laser height metrics considered. High correlations between laser height metrics, but derived from full waveform data, were also reported by Lefsky et al. (1999). From the laser height metrics considered, the linear model using the mean height of all laser returns within a plot was determined to be the most useful model. It accounted for 53% of the variance in observed above ground biomass and predicted the mean above ground biomass to within 2% of the mean value calculated from the ground reference data. More recently, Nelson et al. (2004) demonstrated that by using line intercept sampling, a custom, portable and inexpensive airborne laser profiling instrument (Nelson 2003), and the simulation and modelling approached described by Nelson (1997) and Nelson et al. (1997), the total above ground biomass of forests in the State of Delaware, USA, could be estimated within 19% and 16% of those estimates made by the United States Forest Service at the county and state level, respectively.

The application of discrete return laser scanning for the estimation of above ground biomass for a deciduous forest ecosystem, composed predominantly of sugar maple (*Acer saccharum* Marsh.) and yellow birch (*Betula alleghaniensis* Britton), was explored by Lim et al. (2003*a*). Above ground biomass was estimated using linearly transformed multiplicative models with the height corresponding to the maximum laser return ($r^2 = 0.82$), the mean height of all laser returns ($r^2 = 0.78$) and the mean height of all laser returns that exceeded an arbitrary intensity value threshold ($r^2 = 0.85$) within sample plots as individual predictor variables (Lim et al. 2003*a*). Additional studies reporting on the capabilities of discrete return airborne laser scanning for the estimation of above ground biomass in conifer and mixed forests are required.

*Conceptual model*

Few studies to date have compared the vertical distribution of laser canopy heights acquired with LIDAR remote sensing with the vertical distribution of leaf (or needle) area derived from direct measurements of forest canopies. Consequently, the relationship between the two vertical distributions is not well understood for most tree species. An exception is the work by Magnussen & Boudewyn (1998); the focus of their work was on the derivation of stand heights from discrete return laser scanner data using canopy-based quantile ($q$) estimators for Douglas fir on Vancouver Island, Canada.

A key finding in the study by Magnussen & Boudewyn (1998) was that the distribution of laser canopy heights was a function of the vertical distribution of needle (or leaf) area. More specifically, the two distributions were related to one another following a simple quantile–quantile relationship. Of note is that the relationship between the vertical distributions of laser canopy heights and leaf area, and the ensuing conclusions from the reported relationship, can be extended to leaf mass (i.e. the distribution of leaf area presented by Magnussen & Boudewyn 1998 could have readily been transformed into a distribution of leaf mass using known leaf weight ratios, which is the leaf mass per unit leaf area, for Douglas fir).

If laser height metrics are selected so as to correspond to the same quantile of the distributions of laser canopy heights, and the distributions of laser canopy heights and leaf area and mass are assumed to follow a simple quantile–quantile relationship, then it follows that laser height metrics selected to correspond to a quantile of the distributions of laser canopy heights are estimating some proportion of total leaf area and mass. However, it is well known that above ground biomass is highly correlated with its individual components (e.g. foliar biomass) in addition to leaf area being highly correlated with leaf mass (Burton et al. 1991, Roderick & Cochrane 2002). If the total leaf area and mass are correlated with above ground biomass, then constant proportions of each would also be correlated with above ground biomass. Therefore, it may be postulated that canopy heights corresponding to a quantile of the distributions of laser canopy heights are predictors of above ground biomass and individual components of biomass because they consistently estimate some proportion of leaf area and mass, which are themselves highly correlated with above ground biomass and its components. Therefore, if one canopy-based quantile estimator is found to be correlated with above ground biomass, then any other canopy-based quantile estimator should be equally capable of estimating above ground biomass and components of above ground biomass, provided the allometry of the trees considered is consistent.

### *Objective of study*

The objective of this study is to determine whether the predictive capabilities of models based on different canopy-based quantile estimators (i.e. height metrics that correspond to a quantile of the distributions of laser heights from the forest canopy) are comparable with respect to the estimation of above ground total biomass and individual components of above ground biomass, including stem wood, stem bark, live branch and foliage biomass. To explore this objective, it is assumed that the simple quantile–quantile relationship between the vertical distributions of laser canopy heights and leaf area reported by Magnussen & Boudewyn (1998) can be generalized to other tree species.

## MATERIALS AND METHODS

### *Study site*

The Turkey Lakes watershed (TLW) (47°03′ N, 84°25′ W) is located in the Algoma District, northern Ontario, approximately 60 km north of Sault Ste. Marie, Ontario, Canada, and 13 km inland from Batchawana Bay on Lake Superior (Fig. 1). The TLW is 1000 ha in area and is located at the northern fringe of the Great Lakes–St. Lawrence forest region on the Canadian Shield. The TLW is characterized by an uneven-aged, mature to overmature, old growth hardwood forest predominantly composed of sugar maple (*A. saccharum* Marsh.) and yellow birch (*B. alleghaniensis* Britton). The ages of the primary forest stands throughout the TLW are thought to be 120 yrs and older (Morrison 1990). In 1997, the Turkey Lakes Harvesting Impacts Project (TLHIP) was implemented in the TLW to compare different silvicultural treatments in terms of their impact on sustainable forest management. The silvicultural treatments were applied in a randomized block design and included clearcut, selection, shelterwood and uncut control.

The estimation of various forest biophysical properties (e.g. height and volume) in the TLW was studied previously by Lim et al. (2003*a*). While the diameter at breast height (DBH) and airborne laser scanner data are used again in this study, the objective of this paper and the research carried out differ from previous efforts reported, as do the methods used and laser height metrics examined.

### *Ground reference data*

During the first 2 weeks of July 2000, ground reference data were collected for 36 circular sample plots that were randomly distributed throughout and adjacent to the silvicultural treatment blocks. Within the clear-cut, selection, shelterwood and uncut control treatments, a total of 11, five, nine and 11 sample plots were established, respectively. The distribution of sample plots within each treatment block is shown in Fig. 2. Each sample plot was 0.04 ha (400 $m^2$) in area. Only trees that were greater than 9 cm DBH in each sample plot were sampled. The area of each sample plot and the DBH threshold of 9 cm were selected in accordance with Canada's National Forest Inventory ground sampling protocol. The DBH of trees meeting the sampling criterion was measured with a DBH tape and recorded. Sample plots were georeferenced using post-differentially corrected GPS data. The nominal accuracy of the location of samples plots was estimated to be approximately 2–5 m. Summary statistics stratified by silvicultural treatment for DBH measurements are reported in Table 1.

In addition to above ground total biomass, the biomass allocated to the stem wood, stem bark, live branch and foliage was estimated for each sample plot using site-specific allometric equations (Table 2). When site-specific allometric equations were not available for less frequent tree species in the TLW but found within a sample plot, other equations were substituted with the contribution of these less frequent species to the overall estimates of above ground biomass assumed to be negligible. Substitutions followed recommendations made by I. K. Morrison (Great Lakes Forestry Centre, Canadian Forest Service, Natural Resources Canada, pers. comm., 2002). The sugar maple equation was substituted for red maple, the balsam fir equation for white cedar, and the yellow birch equations for all other species. Plot-level summary statistics stratified by silvicultural treatment for estimates of above ground total, stem wood, stem bark, live branch and foliage biomass are presented in Table 3.

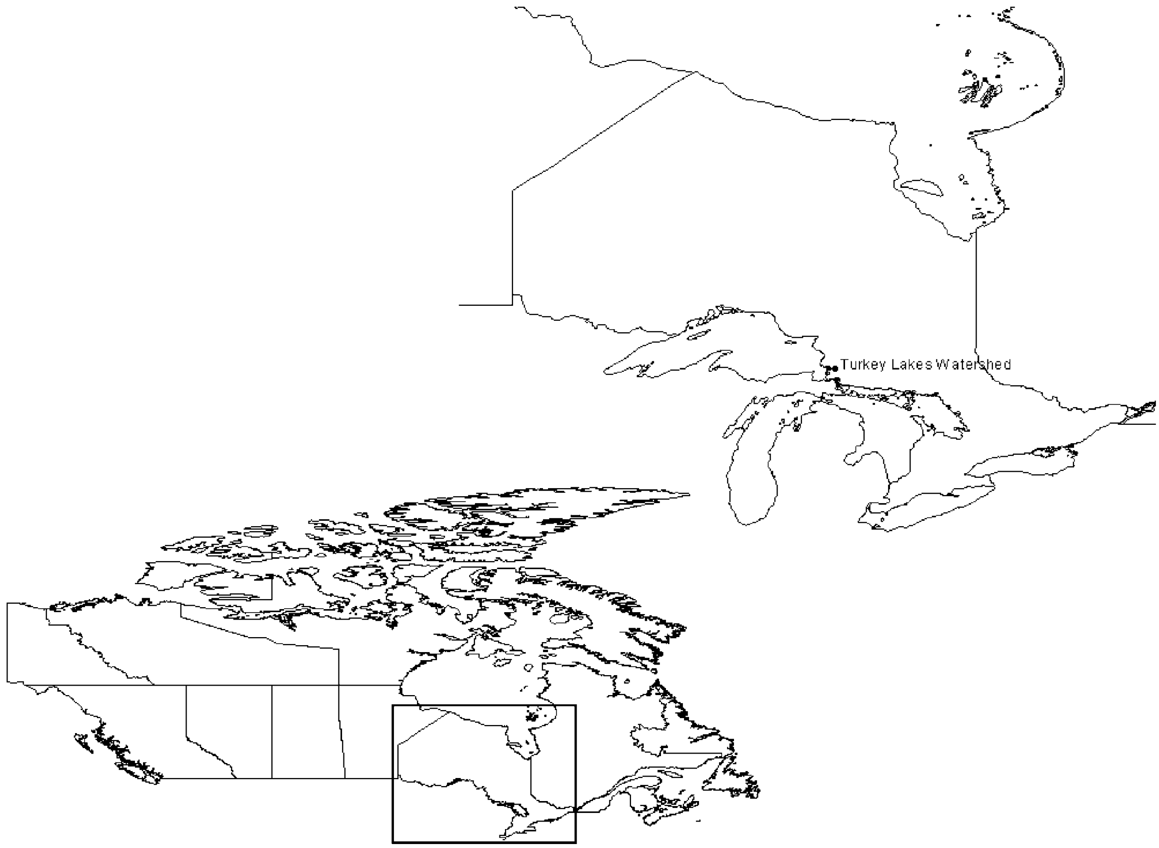


*Fig. 1*. Geographic location of the Turkey Lakes watershed in the Province of Ontario, Canada.

*Airborne laser scanner data*

Airborne discrete return laser scanner data were acquired in late August in 2000 using an Optech Airborne Laser Terrain Mapper (ALTM) 1225 (Optech, Toronto, Ontario, Canada). The ALTM 1225 is capable of a pulse repetition frequency of 25 kHz and was configured with a scanning frequency of 15 Hz, a scan range of $\pm 15°$, and a collection mode of first and last returns, and intensity returns from a 1047 nm laser. The aircraft mounted with the ALTM carried out the survey at 750 m above ground level (AGL) while flying at a velocity of 60 m $s^{-1}$, which resulted in a survey with a swath width of 400 m and a footprint size of approximately 20 cm. The TLW was surveyed using 25% overlapping flight lines and the treatment blocks were surveyed with a second set of flight lines to increase the density of laser point measurements. For the treatment blocks, the average

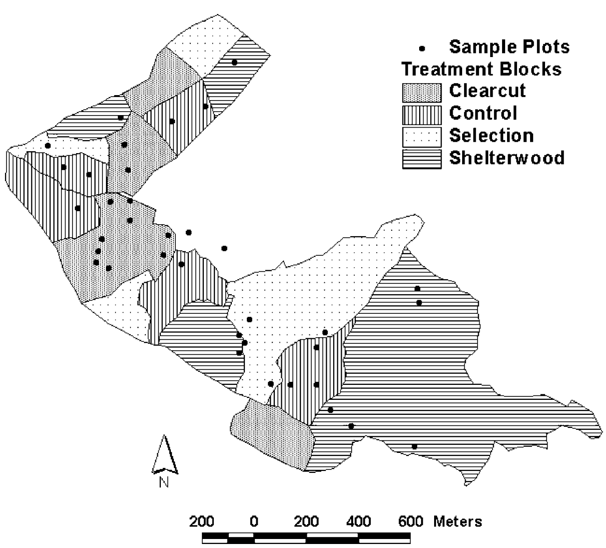


*Fig. 2*. Distribution of sample plots within each silvicultural treatment block.

Table 1. *Summary statistics of the DBH (cm) of trees stratified by silvicultural treatment*

| Treatment | Mean (min.–max.) | No. of plots |
|---|---|---|
| Clearcut | 15.6 (9.1–29.5) | 11 |
| Selection | 25.2 (10.4–49.7) | 5 |
| Shelterwood | 21.4 (9.0–79.5) | 9 |
| Control | 24.2 (9.6–81.6) | 11 |

Table 2. *Site-specific biomass equations, their coefficients of determination and the corresponding number of trees and range of value of DBH (cm) of trees used in destructive sampling for equation development*[a]

| Species | Equation (kg) | $r^2$ |
|---|---|---|
| Sugar maple ($n = 34$, DBH = 2.5–55.2) | | |
| Total (*BIO*) | 0.178 DBH $^{2.340}$ | 0.990 |
| Stem wood (*SW*) | 0.124 DBH $^{2.305}$ | 0.985 |
| Stem bark (*SB*) | 0.023 DBH $^{2.222}$ | 0.979 |
| Live branches (*LB*) | 0.009 DBH $^{2.825}$ | 0.938 |
| Foliage (*F*) | 0.032 DBH $^{1.661}$ | 0.791 |
| Yellow birch ($n = 35$, DBH = 1.6–72.0) | | |
| Total (*BIO*) | 0.144 DBH $^{2.353}$ | 0.996 |
| Stem wood (*SW*) | 0.103 DBH $^{2.298}$ | 0.988 |
| Stem bark (*SB*) | 0.014 DBH $^{2.283}$ | 0.985 |
| Live branches (*LB*) | 0.016 DBH $^{2.621}$ | 0.939 |
| Foliage (*F*) | 0.006 DBH $^{2.159}$ | 0.941 |
| White spruce ($n = 45$, DBH = 10.0–32.4) | | |
| Total (*BIO*) | 0.081 DBH $^{2.450}$ | 0.947 |
| Stem wood (*SW*) | 0.030 DBH $^{2.590}$ | 0.932 |
| Stem bark (*SB*) | 0.006 DBH $^{2.423}$ | 0.874 |
| Live branches (*LB*) | 0.019 DBH $^{2.282}$ | 0.683 |
| Foliage (*F*) | 0.017 DBH $^{2.266}$ | 0.505 |
| Balsam fir ($n = 77$, DBH = 1.4–27.7) | | |
| Total (*BIO*) | 0.175 DBH $^{2.153}$ | 0.949 |
| Stem wood (*SW*) | 0.060 DBH $^{2.304}$ | 0.940 |
| Stem bark (*SB*) | 0.028 DBH $^{1.951}$ | 0.894 |
| Live branches (*LB*) | 0.030 DBH $^{2.162}$ | 0.810 |
| Foliage (*F*) | 0.074 DBH $^{1.699}$ | 0.658 |

[a] Morrison, I. K. Turkey Lakes watershed biomass and volume equations. 2002 (unpublished).

density of laser points was approximately three points per square metre. The laser data, consisting of all laser returns, were classified into ground and vegetation returns by the LIDAR service provider using the Optech REALM 2.27 software (Optech).

*Laser data processing*

The laser data classified as ground returns were interpolated using the inverse distance weighted (IDW) technique (12 nearest neighbours by Euclidean distance; power of 2) to produce a digital elevation model (DEM) of the TLW with a grid cell size of 1 m. The IDW interpolator was chosen following a recommendation by Lloyd & Atkinson (2002) that less complex interpolators be used, such as the IDW interpolator, for the generation of DEMs when the sample spacing is small, which is generally the case with airborne laser scanner data. A comparison of the DEM with point elevation data collected along two transects in the field using traditional surveying techniques ($n = 47$) resulted in an RMSE of 0.27 m. Vegetation heights for laser returns that were not classified as ground were obtained simply by subtracting the DEM $z$-value from the vegetation return's $z$-value at corresponding $x-y$ coordinates. The vegetation class is assumed to consist of laser returns from the forest canopy and understorey vegetation.

*Extracting laser canopy returns*

Based on the assumption that the distributions of laser canopy heights and leaf area in the forest canopy are related, the laser returns comprising the vegetation class must be further segmented into forest and non-forest canopy (i.e. understorey) classes. Distributions of laser heights are derived from the laser returns that are classified as forest canopy, as opposed to all non-ground laser returns in each sample plot. However, classifications based on height thresholds (e.g. excluding laser points falling below a 10 m threshold) were avoided as they presume some prior knowledge of the distribution of tree heights characterizing the sample plots. Instead, a segmentation algorithm using expectation maximization (EM) (Dempster et al. 1977) is used to statistically separate the laser returns corresponding to the forest canopy from non-canopy laser returns (Magnussen & Boudewyn 1998).

The premise of the EM algorithm as it is used here is to fit two linear models to an assumed mixed distribution of laser points and iterate a two-staged

Table 3. *Plot-level summary statistics [mean (min.–max.)] stratified by silvicultural treatment for estimates of aboveground total biomass and components of biomass (Mg ha$^{-1}$)*

| Treatment | Aboveground total biomass | Stem wood | Stem bark | Live branch | Foliage |
|---|---|---|---|---|---|
| Clear-cut | 20.54 (2.20–54.81) | 12.91 (1.38–34.24) | 1.85 (0.18–4.85) | 4.28 (0.46–12.46) | 0.51 (0.06–1.27) |
| Selection | 116.95 (52.70–203.63) | 71.76 (32.73–123.72) | 9.82 (4.59–16.71) | 31.92 (12.19–57.51) | 2.09 (1.17–3.56) |
| Shelterwood | 141.48 (79.76–226.82) | 83.26 (49.75–133.53) | 11.75 (7.02–17.93) | 39.71 (19.10–68.28) | 2.59 (1.81–4.56) |
| Control | 202.14 (131.15–271.08) | 123.28 (81.34–166.02) | 16.74 (11.33–22.44) | 58.27 (33.36–82.72) | 3.55 (2.50–5.24) |

process until the parameters of each linear model converge to a stable solution. The mixed distribution of laser points is assumed to be composed of canopy and non-canopy laser returns. The iteration process consists of two steps: an expectation (E) and a maximization (M) step. During the E-step, for each data point, two weights are calculated, with both weights summing to 1. The weights ($w_i$) are calculated using a function (eqs 1, 2) that uses as input the residuals ($r_i$) from two linear models with known parameters (eqs 3, 4). Note that the function also contains a noise parameter ($\sigma$), which represents the amount of residual expected in the data. The model parameters are calculated during the M step using weighted least squares (eq. 5). Data points are assigned to the model with the minimal residual value. The E and M steps are iterated until model parameters converge.

$$w_1(i) = \frac{e^{-r_1^2(i)/\sigma^2}}{e^{-r_1^2(i)/\sigma^2} + e^{-r_2^2(i)/\sigma^2}} \quad (1)$$

$$w_2(i) = \frac{e^{-r_2^2(i)/\sigma^2}}{e^{-r_1^2(i)/\sigma^2} + e^{-r_2^2(i)/\sigma^2}} \quad (2)$$

$$r_1(i) = a_1 x_i + b_1 - y_i \quad (3)$$

$$r_2(i) = a_2 x_i + b_2 - y_i \quad (4)$$

$$\begin{pmatrix} \sum_i w_i x_i^2 & \sum_i w_i x_i \\ \sum_i w_i x_i & \sum_i w_i 1 \end{pmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_i w_i x_i y_i \\ \sum_i w_i y_i \end{bmatrix} \quad (5)$$

By applying the EM algorithms to all sample plots at once, under conditions where laser canopy heights vary geographically, there is the risk of truncating laser canopy heights from sample plots characterized by shorter trees. Consequently, the EM algorithm was used on an individual plot basis so as to differentiate the forest canopy returns from the other vegetation returns. For each sample plot, 10 iterations were required on average for the parameter values to converge. An example of the EM algorithm applied to a randomly selected sample plot is illustrated in Fig. 3, where height measures on the x–y axis are referenced to the ellipsoid.

### *Statistical analysis*

A multiplicative model formulated as eq. (6) was selected, which can be translated into linear form according to eq. (7). This type of model has been used successfully by others to model various forest biophysical properties (e.g. Means et al. 1999, Næsset 2002, Lim et al. 2003*a*). The natural logarithm transformations required in eq. (7) also ensure that, in most cases, regression assumptions are not violated. Aboveground total biomass and its components are used as the dependent variable, whereas the laser height metrics corresponding to the 0th, 25th, 50th, 75th and 100th percentiles of the distributions of laser canopy heights are the independent variables. Consequently, a total of five linear models was derived. The assumption of normality of error terms and constancy of error variances for each model were assessed using the Shapiro–Wilk and the modified Levene's test, respectively (Neter et al. 1996). Comparisons between models were based on their predictive capabilities with respect to the coefficient of determination ($r^2$) and RMSE of each model. A correction factor, equivalent to the antilog of half the sample variance, was applied to all prediction values so as to account for the bias introduced from log transformations (Baskerville 1972).

$$M = \beta_0 L^{\beta_1} \quad (6)$$

$$\ln M = \ln \beta_0 + \beta_1 \ln L \quad (7)$$

where $M$ is the forest metric of interest, $L$ is some laser height corresponding to a given quantile, and $\beta_0$ and $\beta_1$ are regression coefficients.

## RESULTS

The results from regressing above ground total, stem wood, stem bark, live branch and foliage biomass against the five canopy-based quantile estimators considered (i.e. 0th, 25th, 50th, 75th and 100th percentiles) are summarized in Table 4. The regression coefficients for all models tested were significant at the 0.001 level. The exclusion of clear-cut sample plots when required from the various models considered is noted in the sample size column (i.e. $n$). The results from formal testing for normality of error terms and constancy of error variances for each model are also reported. In general, for each dependent variable considered, the models derived and based on different canopy-based quantile estimators, although not identical, were not overtly different from one another (Table 4).

For the models based on the 100th percentile of the distribution of laser canopy heights, no exclusion of sample plots was required. For all models using height metrics corresponding to the median, and first and third quartile of the distributions of laser canopy heights, one sample plot, which corresponds to a clearcut sample plot (plot 12K), did not follow the observed linear relation. Moreover, its inclusion in

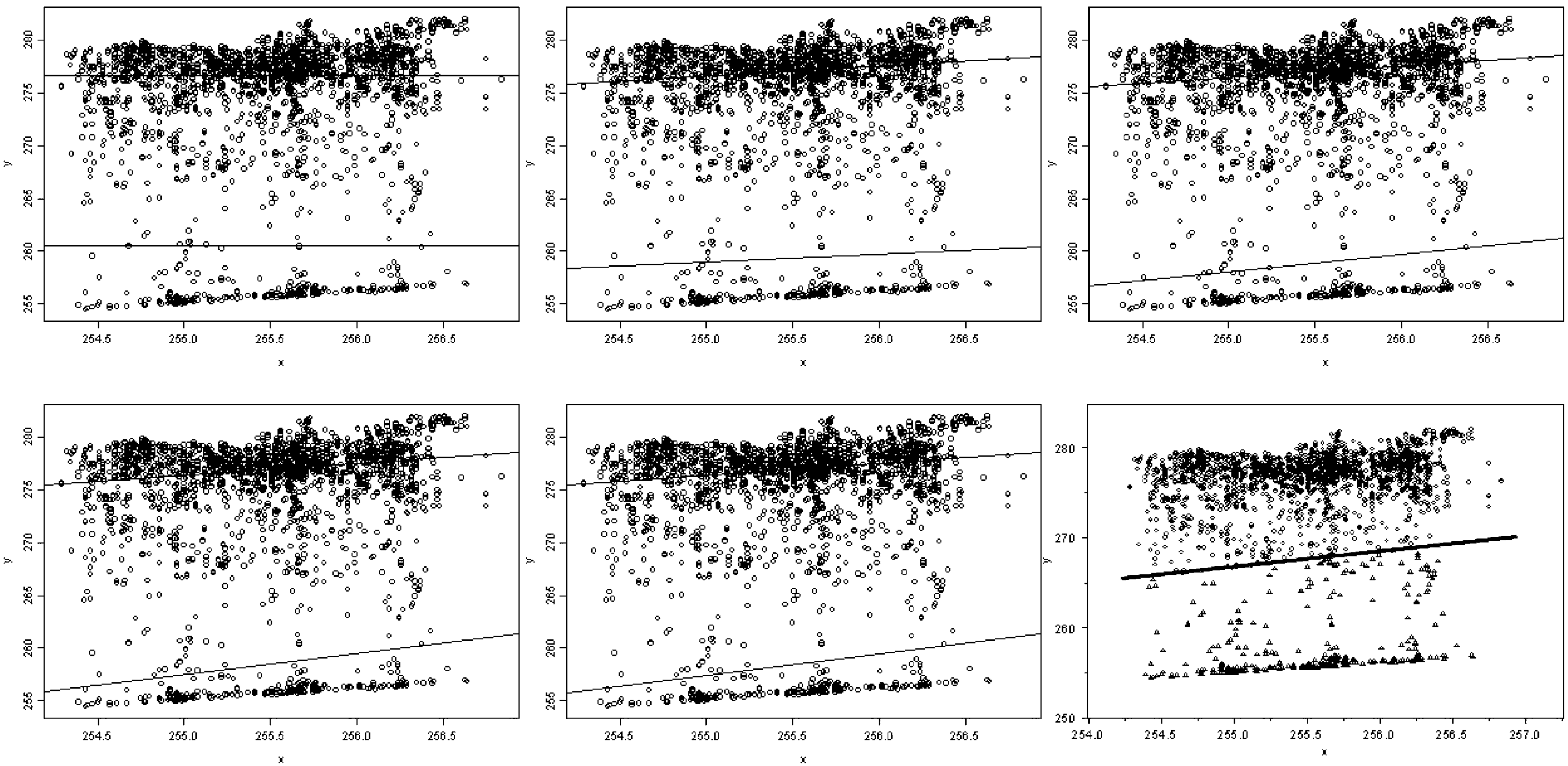


*Fig. 3.* Example of the expectation maximization algorithm applied to a randomly selected sample plot for classification of laser canopy returns from all vegetation returns. Vegetation heights ($y$-axis) are plotted against the $z$-value on the digital elevation model matching its $x$–$y$ coordinates ($x$-axis). Heights are referenced to the ellipsoid in metres. The first, third and last iterations correspond to the top left, top right and bottom centre graphs, respectively. The bottom right graph depicts the results of the segmentation with laser canopy returns symbolized as circles above the solid line.

Table 4. *Summary of models based on different canopy-based quantile estimators used for the estimation of aboveground total, stem wood, stem bark, live branch and foliage biomass*

| | | | | | | | | | Shapiro–Wilk test | | Modified Levene's test | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dependent | Independent | Model | $\ln(\beta_0)$ | $\beta_1$ | $r^2$ | $n$ | RSE | RMSE (Mg ha$^{-1}$) | W | $p$-value | $\lvert t_L{*}\rvert$ | $p$-value |
| BIO | P100 | $\ln(\text{BIO}) = \ln(\beta_0) + \beta_1 * \ln(\text{P100})$ | −11.92* | 5.35* | 0.820 | 36 | 0.537 | 56.16 | 0.945 | 0.075 | 0.518 | 0.608 |
| | P75 | $\ln(\text{BIO}) = \ln(\beta_0) + \beta_1 * \ln(\text{P75})$ | −6.89* | 4.00* | 0.839 | 35[a] | 0.496 | 48.07 | 0.974 | 0.575 | 0.342 | 0.734 |
| | P50 | $\ln(\text{BIO}) = \ln(\beta_0) + \beta_1 * \ln(\text{P50})$ | −6.91* | 4.16* | 0.888 | 34[ab] | 0.390 | 54.00 | 0.981 | 0.816 | 0.204 | 0.840 |
| | P25 | $\ln(\text{BIO}) = \ln(\beta_0) + \beta_1 * \ln(\text{P25})$ | −5.06* | 3.69* | 0.899 | 34[ab] | 0.370 | 50.17 | 0.976 | 0.646 | 0.995 | 0.327 |
| | P0 | $\ln(\text{BIO}) = \ln(\beta_0) + \beta_1 * \ln(\text{P0})$ | −3.92* | 3.83* | 0.833 | 35[b] | 0.491 | 66.65 | 0.964 | 0.299 | 0.731 | 0.470 |
| SW | P100 | $\ln(\text{SW}) = \ln(\beta_0) + \beta_1 * \ln(\text{P100})$ | −12.23* | 5.30* | 0.820 | 36 | 0.530 | 34.14 | 0.945 | 0.072 | 0.580 | 0.566 |
| | P75 | $\ln(\text{SW}) = \ln(\beta_0) + \beta_1 * \ln(\text{P75})$ | −7.26* | 3.96* | 0.840 | 35[a] | 0.489 | 29.04 | 0.975 | 0.602 | 0.351 | 0.728 |
| | P50 | $\ln(\text{SW}) = \ln(\beta_0) + \beta_1 * \ln(\text{P50})$ | −7.23* | 4.11* | 0.889 | 34[ab] | 0.383 | 32.28 | 0.981 | 0.788 | 0.048 | 0.962 |
| | P25 | $\ln(\text{SW}) = \ln(\beta_0) + \beta_1 * \ln(\text{P25})$ | −5.45* | 3.65* | 0.901 | 34[ab] | 0.361 | 29.71 | 0.979 | 0.723 | 0.879 | 0.386 |
| | P0 | $\ln(\text{SW}) = \ln(\beta_0) + \beta_1 * \ln(\text{P0})$ | −4.32* | 3.79* | 0.834 | 35[b] | 0.483 | 40.56 | 0.961 | 0.248 | 0.752 | 0.457 |
| SB | P100 | $\ln(\text{SB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P100})$ | −14.02* | 5.23* | 0.823 | 36 | 0.519 | 4.59 | 0.945 | 0.071 | 0.666 | 0.510 |
| | P75 | $\ln(\text{SB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P75})$ | −9.10* | 3.91* | 0.840 | 35[a] | 0.482 | 6.82 | 0.977 | 0.645 | 0.381 | 0.706 |
| | P50 | $\ln(\text{SB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P50})$ | −9.10* | 4.06* | 0.890 | 34[ab] | 0.377 | 7.37 | 0.978 | 0.717 | 0.261 | 0.796 |
| | P25 | $\ln(\text{SB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P25})$ | −7.31* | 3.61* | 0.903 | 34[ab] | 0.353 | 7.50 | 0.979 | 0.750 | 0.680 | 0.501 |
| | P0 | $\ln(\text{SB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P0})$ | −6.19* | 3.74* | 0.836 | 35[b] | 0.474 | 5.32 | 0.960 | 0.229 | 0.847 | 0.403 |
| LB | P100 | $\ln(\text{LB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P100})$ | −15.05* | 5.93* | 0.808 | 36 | 0.618 | 41.17 | 0.957 | 0.167 | 0.484 | 0.631 |
| | P75 | $\ln(\text{LB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P75})$ | −9.50* | 4.44* | 0.828 | 35[a] | 0.572 | 39.59 | 0.970 | 0.430 | 0.245 | 0.808 |
| | P50 | $\ln(\text{LB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P50})$ | −9.71* | 4.68* | 0.886 | 34[ab] | 0.443 | 40.67 | 0.981 | 0.815 | 0.197 | 0.845 |
| | P25 | $\ln(\text{LB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P25})$ | −7.60* | 4.14* | 0.893 | 34[ab] | 0.430 | 41.20 | 0.976 | 0.630 | 0.813 | 0.422 |
| | P0 | $\ln(\text{LB}) = \ln(\beta_0) + \beta_1 * \ln(\text{P0})$ | −6.31* | 4.29* | 0.828 | 35[b] | 0.560 | 41.68 | 0.970 | 0.445 | 0.902 | 0.374 |
| F | P100 | $\ln(\text{F}) = \ln(\beta_0) + \beta_1 * \ln(\text{P100})$ | −13.83* | 4.69* | 0.830 | 36 | 0.454 | 0.87 | 0.941 | 0.054 | 0.729 | 0.471 |
| | P75 | $\ln(\text{F}) = \ln(\beta_0) + \beta_1 * \ln(\text{P75})$ | −9.34* | 3.48* | 0.841 | 35[a] | 0.429 | 1.88 | 0.970 | 0.433 | 0.155 | 0.877 |
| | P50 | $\ln(\text{F}) = \ln(\beta_0) + \beta_1 * \ln(\text{P50})$ | −9.04* | 3.50* | 0.860 | 34[ab] | 0.373 | 1.92 | 0.960 | 0.250 | 0.139 | 0.891 |
| | P25 | $\ln(\text{F}) = \ln(\beta_0) + \beta_1 * \ln(\text{P25})$ | −7.45* | 3.11* | 0.872 | 34[ab] | 0.356 | 2.00 | 0.960 | 0.238 | 1.452 | 0.156 |
| | P0 | $\ln(\text{F}) = \ln(\beta_0) + \beta_1 * \ln(\text{P0})$ | −6.62* | 3.27* | 0.822 | 35[b] | 0.436 | 1.06 | 0.955 | 0.162 | 0.600 | 0.553 |

[a] Clear-cut plot 12K excluded.
[b] Clear-cut plot 6K excluded.
RMSE: root-mean-square error; *BIO*: total biomass; *SW*: stem wood; *SB*: stem bark; *LB*: live branch; *F*: foliage.
* Significant at $\alpha = 0.001$.

each model resulted in the inability to satisfy the regression assumption of normality of error terms and it was therefore excluded from the analysis so that statistical inferences could be made. For all models using height metrics corresponding to the median and first quartile of the distributions of laser canopy heights, clearcut sample plot 6K was characterized as an outlier and excluded from the analysis as it noticeably did not follow the observed linear relation. In the case of all models based on the 0th percentile of the distribution of laser canopy heights, the clearcut sample plot 6K was excluded since its corresponding laser canopy height of 0 m was not a measure of canopy height.

For all models considered across all dependent variable groups, the laser height metrics corresponding to a consistent quantile of the distributions of laser canopy heights were highly correlated with above ground total biomass and its components (i.e. each $r^2$ greater than 0.80). The largest difference between models with respect to the amount of variance in observed values accounted for by each model was 0.08 for above ground total, stem wood and stem bark biomass, 0.09 for live branch biomass and 0.05 for foliage biomass. With respect to the RMSE of each model within each dependent variable group, differences in RMSE for above ground total, stem wood, stem bark, live branch and foliage biomass were 18.6, 11.5, 2.9, 2.1 and 1.1 Mg ha$^{-1}$, respectively. From these RMSE results, the predictive capabilities of models for above ground total and stem wood biomass appear to be more variable (i.e. the largest RMSE of 66.65 and 40.56 Mg ha$^{-1}$ for above ground total and stem wood biomass correspond to the model based on the 0th percentile). If the model based on the 0th percentile is excluded, the largest difference in RMSE for models estimating above ground total and stem wood biomass is 8.1 and 5.1 Mg ha$^{-1}$, respectively. A pattern where certain models based on a given quantile consistently outperformed other models was not observed.

With the exception of the models based on the 100th percentile, or maximum laser height within sample plots, all other models required the exclusion of at least one, and in some instances two, of the clearcut sample plots (i.e. plots 12K and 6K). An examination of the ground reference data reveals that clearcut sample plot 12K was populated by three trees with DBH measurements of 9.1, 18.8 and 20.9 cm, whereas only a single tree with a recorded DBH of 21.2 cm was found on clearcut sample plot 6K. The number of trees observed on other clearcut sample plots ranged from two to 16 trees with varying DBH.

## DISCUSSION

While the assumption may be valid that the laser sampling point density is sufficient to characterize forest canopy elements, it may be questionable when characterizing individual tree canopies in the case of the clearcut plots. Moreover, the accuracy of georeferenced sample plots with fewer trees becomes more of an issue as inaccuracies in the location of sample plots could result in the exclusion of trees. The consequence of inadvertently excluding trees from sample plots that are already populated by very few trees is that the distributions of laser canopy heights derived may not in fact be representative of those sample plots.

In the case of plot 12K, it was likely that a combination of both the accuracy of the georeferenced sample plots and laser sampling point density affected the distribution of laser canopy heights. However, in the case of clearcut sample plot 6K, considering that the height corresponding to the 0th percentile is 0 m, it is evident that this height is not representative of the forest canopy, but instead of the ground, and that the laser sampling point density was not sufficient for characterizing this sample plot. Therefore, forest canopy cover and laser sampling point density are two factors that may affect the use of canopy-based quantile estimators as they can invalidate prior assumptions regarding to the relationship between the vertical distributions of laser canopy heights and leaf area. In general, additional errors to all models developed could have been introduced as a result of the inaccuracies associated with the georeferencing of sample plots, but such errors are difficult to assess.

The use of the EM algorithm relies on the assumption that some of the laser returns were misclassified as vegetation instead of ground. The actual canopy boundary is determined from the assignment of laser data points to the model that minimizes its residual, which is determined by an iterative process. A potential problem that can arise from using the EM algorithm occurs when few laser returns are misclassified as vegetation and consequently there are too few laser points to which a linear model can be fitted. This situation can arise when the forest canopy is very dense or when the laser sampling point density is too low, both of which lead to a reduced probability of penetration to the low-lying vegetation and ground. However, under these circumstances, laser canopy boundaries are generally clearly defined and there is no confusion as to what height should be used as the canopy boundary. An attempt to compare the canopy boundaries defined by the EM algorithm with measurements of the average height to the crown base of trees in sample plots would not be plausible, given that it assumes that the laser pulses penetrate to the full depth of the canopy. As a result, expectations that canopy boundaries as defined by the EM algorithm should match those measured in the field are not well founded.

Lim et al. (2003*a*) demonstrated previously for the TLW that total above ground biomass could be estimated from the maximum laser return, the mean height of all laser returns and the mean height of all laser returns that exceeded an arbitrary intensity value threshold within sample plots. This research demonstrates that besides those laser height metrics considered by Lim et al. (2003*a*), total above ground biomass and individual components of biomass can be estimated using various canopy-based quantile estimators, with all canopy-based quantile estimators considered comparable with respect to their predictive capabilities. This finding can be attributed to the high correlation between canopy-based quantile estimators (i.e. all $r > 0.91$).

An issue that arises from the use of canopy-based quantile estimators for above ground biomass estimation revolves around the generality of canopy-based quantile estimators to other forest ecosystems and site conditions. Recall that the TLW is considered a mature to overmature forest and that the forest canopies are primarily dominated by sugar maple and yellow birch. It is these trees that are being characterized by the laser scanner data and not the younger and smaller trees that are found in the understorey. The allometry for these mature and overmature trees, even across different treatment blocks, can be considered the same. The implementation of different silvicultural treatments changed the forest structure, but probably had negligible effects on the allometry of the trees considering their age and maturity. Therefore, the relationship between leaf area and a proportion of leaf area, as assumed to have been estimated by the laser heights corresponding to a quantile of the distributions of laser canopy heights, and above ground biomass was probably consistent across all silvicultural treatments. The application of canopy-based quantile estimators for above ground biomass estimation in forests characterized by younger and immature trees, where more variable allometric relationships may be present, could result in several different linear relations emerging for those forests and consequently several different linear fits as opposed to a single fit. Moreover, the utility and applicability of canopy-based quantile estimators in mixed forests, typically characterized by multiple canopy layers and more complex forest structures, remain to be assessed. Nonetheless, the results from this paper demonstrate the potential of canopy-based quantiles estimators in applications of above ground biomass estimation in sugar maple.

## CONCLUSIONS

The concept of canopy-based quantile estimators was introduced and formulated from a conceptual model describing the relationship between the vertical distributions of laser canopy heights and leaf area. Canopy-based quantile estimators were applied to the estimation of above ground biomass and individual components, including stem wood, stem bark, live branch and foliage biomass, in an uneven-aged, mature to overmature, tolerant hardwood forest. The results provide initial support for the concept of canopy-based quantile estimators of above ground forest biomass and its components. Models based on laser height metrics corresponding to different, but consistent quantiles of the distributions of laser canopy heights were able to estimate total, stem wood, stem bark, live branch and foliage biomass to similar degrees with respect to their $r^2$ and RMSE. No single model based on the canopy-based quantiles considered for the estimation of total, stem wood, stem bark, live branch and foliage biomass was overtly different from the others within each dependent variable group. It is assumed that the generality of models based on canopy-based quantile estimators from laser data will be restricted to forests for which the allometry of trees is the same; however, it remains to be assessed whether a single quantile of the distributions of laser canopy heights for forests at different ages and maturity can be used for above ground biomass estimation.

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