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Height–diameter equations for boreal tree species in Ontario using a mixed-effects modeling approach

Mahadev Sharma [a,*], John Parton [b,1]

[a] Ministry of Natural Resources, Ontario Forest Research Institute, 1235 Queen St East, Sault Ste Marie, Ont. P6A 2E5, Canada
[b] Ministry of Natural Resources, Timmins, Ont. P0N 1H0, Canada

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Abstract

Height–diameter relationships based on stand characteristics (trees/ha, basal area, and dominant stand height) were investigated for balsam fir, balsam poplar, black spruce, jack pine, red pine, trembling aspen, white birch, and white spruce using data from permanent growth study plots in northern Ontario, Canada. Approximately half the data were used to estimate model parameters with the rest used for model evaluation. Multiple Chapman–Richards functions with parameters expressed in terms of various stand characteristics were fit to determine the best models for predicting height.

Models providing the most accurate prediction of height included basal area, trees/ha, dominant stand height, and diameter at breast height (DBH). A mixed-effects modeling approach was applied in fitting the models for all tree species. Heights predicted by models including random-effects parameters (calibrated response) were compared with those developed without random-effects parameters (fixed-effects response). Including the random parameter consistently resulted in a better fit to the data (smaller AIC values) and an improved prediction accuracy.

Keywords: Nonlinear least squares regression; Mixed-effects modeling; Boreal tree species; Height–diameter equations

1. Introduction

Growth and yield models are generally used to predict the temporal development of forest stands. Knowledge of diameter at breast height (DBH) and total tree height is fundamental to both developing and applying many growth and yield models. DBH of a tree can be measured quickly, easily, and accurately, but the measurement of total tree height is relatively complex, time consuming, and expensive. Furthermore, tree, stand, and site conditions may prevent accurate height measurements on all trees measured for DBH as it may not be possible to unambiguously observe a given tree, or reach an appropriate vantage point. Therefore, with many permanent and temporary sample plot systems, DBH is conventionally measured for all trees sampled, but height is measured for only a sub-sample of trees selected across the range of diameters observed (Huang et al., 1994). Height–diameter relationship models are then used to estimate the heights of trees measured only for diameter.

The development of simple and accurate height–diameter models, based on easily obtainable tree and stand characteristics, is a common precursor to using inventory and sample plot data to calculate volume and other stand attributes. A number of height–diameter equations have been developed using only DBH as the predictor variable for estimating total height (e.g., Curtis, 1967; Wykoff et al., 1982; Larsen and Hann, 1987; Wang and Hann, 1988; Huang et al., 1992; Moore et al., 1996; Zhang, 1997; Peng, 1999; Fang and Bailey, 1998; Fekedulengn et al., 1999; Jayaraman and Zakrzewski, 2001; Robinson and Wykoff, 2004). However, the relation between the diameter of a tree and its height varies among stands (Calama and Montero, 2004) and depends on the growing environment and stand conditions (Sharma and Zhang, 2004).

For a particular height, trees that grow in high density stands will have smaller diameters than those growing in less dense stands, because of greater competition among individuals (Lopez Sanchez et al., 2003; Calama and Montero, 2004). The height–diameter relationship is also not constant over time,

* Corresponding author. Tel.: +1 705 946 7407.
E-mail addresses: mahadev.sharma@mnr.gov.on.ca (M. Sharma), john.parton@mnr.gov.on.ca (J. Parton).
[1] el.: +1 705 235 1238.

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even within the same stand (Curtis, 1967). These factors indicate that additional predictor variables are required to develop generalized height–diameter models in order to avoid having to establish individual height–diameter relationships for every stand (Temesgen and Gadow, 2004).

In the development of generalized height–diameter relationships, several approaches have been described and utilized. For example, Ferguson and Leech (1978), Krumland and Wensel (1978), Larsen and Hann, 1987, and Parresol (1992) first fit a height–diameter model to each individual plot, and then explained the parameters in terms of stand characteristics such as stand density, basal area, dominant height, age, and dominant diameter. Fulton (1999), Huang et al. (2000), and Zhang et al. (2002) developed discrete models for different geographical or ecological regions.

Lappi (1997) and Hökkä (1997) used models with random coefficients defining a fixed population average response and varying random parameters for each plot. Harrison et al. (1986) included stand dominant height in their height–diameter model in order to improve model efficacy. Soares and Tomé (2002) used stand dominant height, maximum diameter, and density as predictor variables in addition to DBH. Lopez Sanchez et al. (2003) and Eerikäinen (2003) used stand dominant height, dominant diameter, density, and age information in their models to improve model accuracy. Similarly, Zakrzewski and Bella (1988) used quadratic mean diameter and the height of tree with quadratic mean diameter to increase model efficiency.

Calama and Montero (2004) reported that covariates such as dominant height, diameter distribution percentiles, and stand density, explained the plot variability in developing height–diameter models for stone pine (*Pinus pinea* L.). Fang and Bailey (1998) also used diameter distribution percentiles in localizing their height–diameter model for tropical forest trees in China. Sharma and Zhang (2004) incorporated stand density (trees/ha and basal area/ha) and site index information in developing height–diameter models for jack pine (*Pinus banksiana* Lamb.) and black spruce (*Picea mariana* (Mill.) B.S.P.) trees. They reported that the Chapman–Richards function with the asymptote and rate parameters expressed in terms of the basal area and trees/ha, respectively, was superior to other models for estimating heights of these two tree species.

The diversity observed in modeling approaches was easily dwarfed by the multiplicity in model forms developed. Numerous linear and nonlinear models have been applied in the quest to produce accurate height–diameter relationships. Candidate functions were often based on their mathematical features (e.g., demonstrating appropriate sigmoidal shape, possessing a sufficient number of parameters to achieve flexibility without compromising parsimony), and the possible biological interpretation of model parameters (Peng et al., 2001). Biologically reasonable models generally produced more accurate predictions beyond the range of data used in model fitting (Fekedulengn et al., 1999).

The province of Ontario holds approximately 70.2 million hectares, or about 17% of Canada's forest, and about 2% of the world's forests (OMNR, 2001). Ontario's Provincial Growth and Yield Program manages an extensive network of permanent sample plots, however, height–diameter models that can be applied to accurately estimate heights of all major commercial tree species making use of plot-specific information are unavailable. The objectives of this study were to (1) develop static height prediction equations in terms of easily measured tree and stand characteristics for eight major commercial tree species grown in Ontario's boreal forests. These tree species are: balsam fir (*Abies balsamea* (L.) (Mill.)), balsam poplar (*Populus balsamifera* L.), black spruce (*Picea mariana* (Mill.) B.S.P.), jack pine (*Pinus banksiana* Lamb.), red pine (*P. resinosa* Ait.), trembling aspen (*Populus tremuloides* Michx.), white birch (*Betula papyrifera* Marsh.), and white spruce (*Picea glauca* (Moench) Voss), (2) determine if the inclusion of a plot and species-specific random-effects parameter improved prediction accuracy relative to the fixed-effects model.

2. Methods

2.1. Data

Data used in this study were obtained from permanent sample plots established and maintained by the Forest Ecosystem Science Co-operative Inc. in Ontario, Canada, and from plots established and maintained by the Ontario Ministry of Natural Resources (OMNR). The plots are located across much of the commercially operable forest of northern Ontario, Canada. Most of the plots used in this study have been measured only once.

The data set contained both plot- and tree-level information. The plot level information included site index calculated from working group species (species with the greatest basal area in the plot), total tree basal area (BA/ha), number of stems (N/ha, calculated from the trees with DBH greater than or equal to 2.5 cm), and the area of the plot (400 m^2). There were 5498 plots used in this study. Mean values of BA/ha, trees/ha, and stand dominant height (calculated from a sample of dominant and codominant trees with measured heights) for these plots were 24.22 m^2, 2236, and 15.21 m with their standard deviations 9.81 m^2, 1583, and 5.21 m, respectively. Ranges of these stand characteristics were 5.03 to 85.32 m^2 for BA/ha, 50–17,625 for trees/ha, and 3.80–33.10 m for dominant stand height.

The tree level information contained tree status (live or dead), origin (natural or planted), species, and DBH, for all the trees, and total height for a subset of trees on a plot. Trees that were dead, defective at DBH, or possessing a broken top were not used in this study. In total, 6400 balsam fir, 1145 balsam poplar, 20,325 black spruce, 20,652 jack pine, 1005 red pine, 13,211 trembling aspen, 8462 white birch, and 5304 white spruce trees were available. Summary statistics for total height and DBH are presented in Table 1.

Since not all plots contained all the species under consideration, species composition (% of basal area of the species under consideration) was calculated for each species in each plot (Table 2). Black spruce and red pine were found to be in the highest (4089) and lowest (167) number of plots, respectively. On the other hand, jack pine and balsam fir contributed average basal areas of 62.6% (highest) and 12.6% (lowest), respectively, in the plots they inhabited. About half of

Table 1
Summary statistics for diameter at breast height (DBH) (cm) and total height (m) of tree species used in fit and validation data sets

Species	Data set	Number of trees	Variable	Mean	S.D.	Minimum	Maximum
Balsam fir	Model	3493	DBH	11.32	6.39	2.50	40.60
			Height	9.43	4.58	1.94	29.67
	Validation	2907	DBH	11.03	6.37	2.50	37.90
			Height	9.27	4.63	2.14	23.86
Balsam poplar	Model	609	DBH	17.18	10.89	2.50	55.80
			Height	14.97	6.40	2.59	29.46
	Validation	536	DBH	17.40	10.70	2.50	55.10
			Height	15.45	6.37	2.50	29.86
Black spruce	Model	11590	DBH	12.97	6.14	2.50	42.90
			Height	11.32	4.76	1.33	27.30
	Validation	8735	DBH	13.25	6.23	2.50	42.90
			Height	11.51	4.72	2.08	30.78
Jack pine	Model	10919	DBH	16.23	7.38	2.50	53.20
			Height	14.04	5.49	2.24	28.67
	Validation	9733	DBH	16.13	7.22	2.50	48.50
			Height	13.92	5.34	2.46	27.80
Red Pine	Model	666	DBH	21.43	11.80	2.50	68.50
			Height	16.26	7.32	2.35	31.17
	Validation	339	DBH	17.57	11.35	2.50	54.60
			Height	13.82	8.07	2.20	32.20
Trembling aspen	Model	7462	DBH	16.87	9.54	2.50	62.20
			Height	16.22	6.45	1.78	36.15
	Validation	5749	DBH	16.73	9.75	2.50	64.80
			Height	15.95	6.41	2.59	37.80
White birch	Model	4529	DBH	11.13	6.62	2.50	45.70
			Height	11.30	4.41	2.68	26.02
	Validation	3933	DBH	11.20	7.08	2.50	52.70
			Height	11.19	4.58	2.11	26.39
White spruce	Model	2853	DBH	14.45	8.70	2.50	59.40
			Height	10.75	5.37	2.01	30.78
	Validation	2451	DBH	14.19	8.55	2.50	51.90
			Height	10.46	5.20	2.19	30.17

the plots were randomly selected for each species and all the trees from these plots were used to estimate model parameters (model data set). The trees in the remaining plots were used for model validation (validation data set).

2.2. Height–diameter models

Tree height and diameter relationships are generally described using nonlinear mathematical models. The most common models used in forestry are Chapman–Richards (Chapman, 1961; Richards, 1959), Weibull (Yang et al., 1978), Schnute (Schnute, 1981), exponential (Ratkowsky, 1990), logistic (Ratkowsky and Reedy, 1986), and Korf (Stage, 1963; Zeide, 1989; Mehtatalo, 2004). In these models, tree height or diameter is usually expressed as a function of tree or stand age. Burkhart and Tennent (1977), Carmean and Lenthall (1988), Goelz and Burk (1992), Tewari and Kumar (2002), and others found the Chapman–Richards function appropriate in describing height development over stand age for a variety of tree species.

Table 2
Summary statistics for species composition (% of the basal area of the species in consideration) in different plots

Species	Number of plots	Mean	S.D.	Minimum	Maximum
Balsam fir	2495	12.58	17.94	0.03	100.00
Balsam poplar	394	14.44	21.12	0.05	100.00
Black spruce	4089	39.40	36.65	0.02	100.00
Jack pine	3207	62.63	34.88	0.04	100.00
Red pine	167	47.26	36.95	0.12	100.00
Trembling aspen	2781	31.45	32.82	0.05	100.00
White birch	2722	13.76	19.20	0.04	100.00
White spruce	1781	19.34	27.89	0.03	100.00

A simplified version of the Chapman–Richards function is:

$$Y = \alpha(1 - e^{-\beta X})^{\gamma} \quad (1)$$

where Y is tree or stand size (e.g., height, diameter, volume), X the independent variable (e.g., age), and α, β, and γ are the asymptote, rate, and shape parameters, respectively. Since height growth increases with increasing site quality, several researchers (e.g., Brickell, 1966; Burkhart and Tennent, 1977; Newnham, 1988; Biging, 1985) expanded Eq. (1) by expressing each model parameter as a function of site index in describing height growth.

Eq. (1) has also been used to describe total tree height as a static function of diameter. Peng et al. (2001), for example, examined the relative performance of the Chapman–Richards model to predict total tree height as a function of DBH for nine boreal forest tree species in Ontario, Canada. They recommended the Chapman–Richards function for these tree species on the basis of its mathematical properties, its amenity to biological interpretation, and its satisfactory prediction performance. The exact form of the Chapman–Richards equation recommended by Peng et al. (2001) for jack pine and black spruce trees was:

$$H = 1.3 + \alpha(1 - e^{-\beta D})^{\gamma} \quad (2)$$

where H = total tree height (m), and D = diameter at breast height (DBH) (cm). The height–DBH relationship of a tree species, however, varies among different environments and stand conditions, e.g., stand density, basal area, and site index (Lopez Sanchez et al., 2003; Calama and Montero, 2004; Sharma and Zhang, 2004). To account for this variation, Sharma and Zhang (2004) expanded Eq. (2) using stand characteristics. They expressed the parameters of this function in terms of stand basal area, stand density, and site index to establish a height–diameter relationship that was sensitive to stand conditions. Comparing several models, they reported that the following models (Eqs. (3)–(5)) were very similar in terms of fit characteristics (mean square error, MSE and coefficient of determination, R^2) for jack pine and black spruce trees:

$$H = 1.3 + \alpha(\mathrm{BA})^{\delta}(1 - e^{-\beta(\mathrm{TPH})^{\varphi} D}) \quad (3)$$

$$H = 1.3 + \alpha(\mathrm{BA})^{\delta}(1 - e^{-\beta(\mathrm{TPH})^{\varphi} D})^{\gamma} \quad (4)$$

$$H = 1.3 + \alpha(\mathrm{BA})^{\delta}(1 - e^{-\beta(\mathrm{TPH})^{\varphi} D})^{\gamma(\mathrm{SI})^{\eta}} \quad (5)$$

where BA = stand basal area (m^2/ha), TPH = stand density (trees/ha), SI = site index (m), α, β, γ, δ, φ, and η are parameters and other variables as defined earlier.

All these models ensure that the total height equals breast height (1.3 m) when DBH = 0. In terms of predictive ability (less bias in estimating heights), however, Eq. (3) performed the best. They further reported that all of these models (Eqs. (3)–(5)) were superior to model (2) in terms of fit characteristics and predictive ability. The model with site index (Eq. (5)), however, did not perform better than Eq. (3) in term of predictive accuracy.

Dominant stand height (mean height of dominant and co-dominant trees) has been used instead of site index in modeling height–diameter relationships for various tree species (see Lopez Sanchez et al., 2003). Therefore, dominant stand height was used in this study, as the site index calculated from the working group species in the plot did not improve model accuracy (Sharma and Zhang, 2004). The mean height of sampled dominant and codominant trees in a plot, irrespective of species, was used as the dominant stand height for the plot. Other potential models obtained by combining dominant stand height with other stand-level variables were also explored for these tree species.

A preliminary analysis indicated that the Chapman–Richards equation with the asymptote and rate parameters expressed in terms of dominant stand height and TPH/BA (a variable analogous to quadratic mean diameter and possessing the base attributes required to adequately describe stand density, Zeide, 2005), respectively, could improve on previously published results for these species. This model can be expressed mathematically as:

$$H = 1.3 + \alpha(\mathrm{SHT})^{\delta}(1 - e^{-\beta(\mathrm{TPH/BA})^{\varphi} D})^{\gamma} \quad (6)$$

where SHT = dominant stand height. Therefore, Eqs. (2)–(4) and (6) were considered as potential height–diameter models and were evaluated for all eight tree species used in this study.

Data used in modeling height–diameter relationships frequently contain measurements of height and diameters from multiple trees from a given sample plot (although individual plots are generally located within different stands). These groups of height–diameter measurements may violate the basic assumption of independence, as multiple observations from a single sampling unit may be highly correlated (Gregoire, 1987; Calama and Montero, 2004).

One approach to dealing with correlated observations is to specify a mixed-effects model. The variability among plots can be modeled by introducing random parameters. Both fixed- and random-effects parameters can then be simultaneously estimated using the mixed-effects approach.

2.3. Nonlinear mixed modeling

Multiple measurements (height–diameter pairs) taken from a given sample plot are not independent and hence are correlated. To deal with these correlated observations, a nonlinear mixed-effects modeling approach can be applied. A general expression for the nonlinear mixed-effects model can be written as:

$$y_{ij} = f(\boldsymbol{\Phi}_i, x_{ij}) + e_{ij}$$

where y_{ij} is the jth observation (tree) of the response variable taken from the ith sampling unit (plot), x_{ij} the jth measurement for the predictor variable at the ith plot, $\boldsymbol{\Phi}_i$ a parameter vector, $r \times 1$ (r is the number of parameters in the model) is specific to each sampling unit, f a nonlinear function of the predictor variables and the parameter vector, and e_{ij} is the random error. In vector form:

$$\mathbf{y}_i = f(\boldsymbol{\Phi}_i, \mathbf{x}_i) + \mathbf{e}_i$$

where, y_i is the $n_i \times 1$ observation vector and $\mathbf{x}_i$ is the $n_i \times 1$ predictor vector for the ith plot, and $\mathbf{e}_i$ is a $n_i \times 1$ vector of the residuals. The parameter vector $\boldsymbol{\Phi}_i$ can then be broken down into fixed and random components. The fixed component is common to all populations (i.e., all sample plots) and the random part varies from plot to plot, i.e.,

$$\boldsymbol{\Phi}_i = \mathbf{A}_i \lambda + \mathbf{B}_i \mathbf{b}_i$$

where λ is the $p \times 1$ vector of fixed population parameters, $\mathbf{b}_i$ is the $q \times 1$ vector of random-effects associated with the ith plot, and $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are design matrices of size $r \times p$ and $r \times q$ for the fixed- and random-effects specific to each plot, respectively. Details on nonlinear mixed effects modeling for height–diameter relationships can be found in the study by Calama and Montero (2004).

Eqs. (2)–(4) and (6) were first fitted to the model data set using ordinary nonlinear least squares regression (NLIN procedure) in SAS (SAS Institute Inc. Cary, NC, 2001). Fit statistics (R^2 and MSE) and bias in predicting heights using these models were calculated for each diameter and density class for both model and validation sets. AIC values were also calculated using NLMIXED procedure for all models for all species. Evaluation consisted of examining fit statistics and prediction accuracy. The model resulting in the largest R^2, least MSE, and smallest values of AIC and average bias and its standard deviation for each diameter and density class was selected as the best model for these tree species. The mixed-effects method was then applied to this 'best' model which was fit using the NLMIXED procedure in SAS. Parameters of the "best" model were estimated with and without the presence of random parameter(s). Fixed-effect parameters estimated in the presence and absence of the random-effect parameter(s) were compared.

2.4. Prediction

The main purpose of developing a model is to predict the dependent variable (*H*, total tree height in this case) in terms of independent variables (TPH, SHT, BA, and DBH) through the relationship specified in the model. In the mixed-effects modeling approach, height can be predicted (i) by assuming the random parameters are zero if no local height information is available (fixed-effects response), (ii) by predicting the random parameters for the plot for which height information is available for a sub-sample of trees (calibrated response).

In the case of a fixed-effects response, the predicted height represents the mean behavior of the pattern of variation in height for both the given diameter and the associated stand characteristics. Therefore, the heights are predicted using the expression:

$$\hat{h}_{ij} = f(\hat{\boldsymbol{\Phi}}_i, x_{ij})$$

where $\hat{\boldsymbol{\Phi}}_i = A_i\hat{\lambda}$, $\boldsymbol{A}_i$ and $\hat{\lambda}$ are design matrix and estimated vector for the fixed effects, respectively. x_{ij} and $\hat{h}_{ij}$ are the vector of the independent variables (SHT, BA, TPH, and D) and predicted height, respectively, for the *j*th tree in the *i*th plot.

In the case of calibrated response, however, model parameters are localized first by using the predicted values of the random parameters for each plot. Height–diameter data from a sub-sample of trees from the plot are used along with stand characteristics to predict the random parameters for the plot. The following expression can be used to predict the random parameters (Vonesh and Chinchilli, 1997):

$$\hat{\boldsymbol{b}}_i = \hat{D}\hat{Z}_i^{\mathrm{T}}(\hat{R}_i + \hat{Z}_i\hat{D}\hat{Z}_i^{\mathrm{T}})^{-1}\hat{\boldsymbol{e}}_i \tag{7}$$

where $\hat{D}$ is the $q \times q$ variance–covariance matrix (q = number of random-effects parameters included in the model) for the among-plot variability, $\hat{R}_i$ is the $k \times k$ variance–covariance matrix for plot i, $\hat{\boldsymbol{e}}_i$ is the residual vector $k \times 1$ with components $\hat{e}_{ij}$ defined as:

$$\hat{e}_{ij} = h_{ij} - f(\hat{\boldsymbol{\Phi}}_i, x_{ij})$$

where h_{ij} = observed height for the *j*th tree in the subsample from plot i, $\hat{\boldsymbol{\Phi}}_i = A_i\hat{\lambda}$ including only the fixed part of the estimated vector of parameters, $\hat{Z}_i$ is the $k \times q$ matrix evaluated at $\hat{\lambda}$ as:

$$\begin{matrix} \dfrac{\partial f(x_{i1}, \hat{\boldsymbol{\Phi}}_i)}{\partial \lambda_1} & \cdots & \dfrac{\partial f(x_{i1}, \hat{\boldsymbol{\Phi}}_i)}{\partial \lambda_q} \\ \cdots & \cdots & \cdots \\ \dfrac{\partial f(x_{ik}, \hat{\boldsymbol{\Phi}}_i)}{\partial \lambda_1} & \cdots & \dfrac{\partial f(x_{ik}, \hat{\boldsymbol{\Phi}}_i)}{\partial \lambda_q} \end{matrix}$$

where, $\lambda_1, \ldots, \lambda_q$ are the fixed part of the mixed coefficients components of the vector for estimated fixed-effects $\hat{\lambda}$, and x_{ij} is the vector of independent variables corresponding to the *j*th tree in the subsample of the *i*th plot. Random-effects predicted in such a way are added to the fixed parameters to obtain localized parameters. Heights of the trees measured only for diameters are then predicted in terms of diameters, stand characteristics, and localized parameters. Details on the prediction of random effects parameters in the forestry context can be found in the study by Calama and Montero (2004) and Trincado and Burkhart (2006).

Prediction accuracies of the models with and without the random-effect were compared by examining the bias and its standard deviation of fixed-effects and calibrated responses for each diameter and density class for the validation data set. Unless otherwise specified, the level of significance is 0.05 (alpha = 5%) throughout this paper.

3. Results and discussion

Table 3 displays the fit statistics (R^2, MSE, and AIC) for Eqs. (2)–(4) and (6) derived from the NLIN and NLMIXED procedures in SAS. Eqs. (6) and (2) had the best and poorest fits, respectively, among these four equations. Eqs. (3) and (4) produced similar fit statistics. Eq. (2) had the largest variation in R^2 and MSE across species. The coefficient of determination (R^2) varied from 0.757 (jack pine) to 0.901 (balsam poplar) and the mean square error (MSE) varied from 2.97 (balsam fir) to 9.55 (red pine). On the other hand, the R^2 and MSE varied from 0.898 (black spruce) to 0.956 (jack pine) and from 1.43 (jack pine) to 2.67 (red pine), respectively, for Eq. (6). Similarly, AIC decreased from Eq. (2)–(6) for all species but the largest decrease was in the case of jack pine trees.

Fit statistics improved significantly across equations for a given species. Increased values of R^2 and decreased values of MSE and AIC were regarded as improved fit statistics. Jack pine had the largest improvement from Eqs. (2)–(6) for all fit statistics. In this case, the R^2 increased by more than 19%, the MSE decreased by more than 80%, and the AIC decreased by

Table 3
Fit statistics (R^{2a}, MSE, and AIC) for Eqs. (2)–(4) and (6) by species

Goodness of fit	Balsam fir	Balsam poplar	Black spruce	Jack pine	Red pine	Trembling aspen	White birch	White spruce
Eq. (2)								
R^2	0.858	0.901	0.834	0.757	0.822	0.857	0.802	0.885
MSE	2.97	4.06	3.76	7.34	9.55	5.94	3.84	3.30
AIC	13730	2587	48243	52752	3397	34472	18960	11515
Eq. (3)								
R^2	0.891	0.922	0.873	0.888	0.911	0.912	0.855	0.912
MSE	2.29	3.23	2.87	3.39	4.78	3.66	2.81	2.56
AIC	12826	2448	45110	44329	2938	30863	17544	10781
Eq. (4)								
R^2	0.895	0.929	0.874	0.902	0.912	0.917	0.857	0.912
MSE	2.20	2.91	2.86	2.96	4.76	3.45	2.79	2.56
AIC	12681	2386	45075	42858	2936	30419	17504	10782
Eq. (6)								
R^2	0.895	0.955	0.898	0.953	0.951	0.953	0.884	0.917
MSE	2.20	1.88	2.32	1.43	2.67	1.97	2.25	2.39
AIC	12680	2119	42659	34889	2551	26242	16541	10592

[a] Computed as (1 − residual sum of squares/corrected sum of squares).

about 34%. On the other hand, white spruce had the least improvement from Eqs. (2)–(6) for all fit statistics. The differences in respective fit statistics between Eqs. (3) and (4) were negligible (Table 3) for all tree species.

To further evaluate Eqs. (2)–(4) and (6), trees from the validation data sets were divided into different DBH and density classes. A DBH class was assigned to every 2 cm interval starting at 1, i.e., DBH class 2 for 1.1–3.0 cm, 4 for 3.1–5.0 cm, and so on. DBHs greater than 29 cm were included in the >30 cm class. Similarly, a density class was assigned to every 500 TPH starting at 1, i.e., density class 250 for TPH $\leq$500, 750 for TPH $>$ 500 and $\leq$1000, and so on. Densities higher than 7000 TPH were included in the >7000 density class.

Average values of height prediction bias (observed − pre-predicted) and corresponding standard deviation were calculated for each DBH and density class for all tree species for the validation data set. The magnitude and nature of the bias and corresponding standard deviation across density and DBH classes computed using a particular equation were similar for all tree species. Therefore, results from jack pine and black spruce have been presented here because these species had the highest (95.6%) and one of the lowest (89.8%) R^2 values, respectively, among the species studied.

Overall, the mean bias and its standard deviation across DBH classes were smallest for Eq. (6) and largest for Eq. (2) (Table 4) with those for Eqs. (3) and (4) falling in between. The same pattern follows for the mean bias and its standard deviation across density classes for both species (Table 5). Therefore, Eq. (6) was chosen as the best height–diameter relationship model for all eight tree species and used for mixed-effects modeling.

3.1. Model construction

Fang and Bailey (2001) listed the following three steps necessary for constructing a mixed model:

- determine parameter effects;
- determine within-plot variance–covariance structure;
- specify between-plot variation.

Gregoire et al. (1995) pointed out that the determination of fixed and random effects parameters in a model is a flexible decision subject to debate. Pinheiro and Bates (1998) suggest that all parameters in the model should first be considered mixed if convergence is possible. Fang and Bailey (2001) suggest that parameters with the high variability and less overlap in confidence intervals obtained by fitting at each individual plot separately should be considered mixed if the convergence is not achieved when considering all the parameters as mixed.

In Eq. (6), parameters α and δ determine the asymptote, β and φ contribute to the rate parameter, and γ is the shape parameter. In the first step, Eq. (6) was fitted assuming all parameters as random using the NLMIXED procedure in SAS. Then, parameters α, β, and γ (parameters in the original Chapman–Richards equation) were assumed as random. In another attempt, two of the three parameters α, β, and γ were assumed random. The model failed to converge in all cases. Finally, one of the five parameters in Eq. (6) was assumed random. All models with one random parameter converged but the one with β and φ as random. However, the model with α as random parameter resulted in the smallest AIC value for all tree species. Therefore, the parameter α in Eq. (6) was assumed as a mixed parameter, i.e., α consists of a random (u_i) and a fixed (θ) part and the random part is specific to each plot and species combination. For the remainder of this paper, the plot represents the plot species combination unless otherwise specified. The expression for the mixed model for Eq. (6) can be written as:

$$h_{ij} = 1.3 + (\theta + u_i)(\mathrm{SHT}_i)^{\delta}(1 - \mathrm{e}^{-\beta(\mathrm{TPH}_i/\mathrm{BA}_i)^{\varphi} D_{ij}})^{\gamma} + e_{ij} \quad (8)$$

$$u_i \sim N(0, \tau)$$

Table 4
Average bias (observed − predicted) and its standard deviation by diameter (DBH) class in predicting total heights of jack pine and black spruce trees using Eqs. (2)–(4) and (6) for validation data sets

DBH class	Number of trees	Mean bias				Standard deviation			
		Eq. (2)	Eq. (3)	Eq. (4)	Eq. (6)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (6)
Jack pine									
2	160	0.870	0.853	−0.617	0.138	0.954	0.896	0.895	0.845
4	419	1.085	1.037	−0.406	0.109	1.340	1.166	1.134	1.041
6	456	0.559	0.898	−0.188	−0.013	1.852	1.268	1.181	1.165
8	650	−0.752	0.624	−0.070	0.010	2.300	1.392	1.270	1.111
10	836	−0.146	0.454	0.047	−0.046	2.735	1.574	1.439	1.207
12	985	−0.497	−0.010	−0.112	−0.115	2.866	1.607	1.500	1.120
14	1072	−0.292	−0.207	−0.078	−0.111	3.138	1.824	1.738	1.302
16	985	−0.387	−0.376	−0.040	−0.055	3.011	1.740	1.660	1.261
18	891	0.094	−0.232	0.195	−0.002	2.902	1.811	1.763	1.212
20	851	0.280	−0.285	0.149	0.002	2.865	1.873	1.830	1.166
22	695	0.599	−0.489	−0.112	−0.096	2.662	1.805	1.772	1.242
24	553	0.208	−0.418	−0.155	−0.032	2.636	2.007	1.946	1.297
26	467	−0.204	−0.430	−0.349	−0.070	2.629	2.105	2.072	1.377
28	297	−0.164	−0.203	−0.393	−0.051	2.424	2.063	2.034	1.208
>30	416	−0.864	−0.054	−0.721	−0.068	3.094	2.796	2.757	1.413
Black spruce									
2	284	0.027	−0.361	−0.181	−0.018	0.580	0.651	0.643	0.670
4	682	0.129	−0.307	−0.135	−0.038	0.891	0.900	0.899	0.925
6	674	0.141	−0.143	−0.018	0.002	1.258	1.108	1.111	1.156
8	794	0.243	0.115	0.185	0.141	1.701	1.367	1.374	1.366
10	981	0.031	0.022	0.040	−0.024	1.923	1.547	1.551	1.458
12	899	−0.106	0.043	0.017	−0.134	2.068	1.724	1.727	1.593
14	1022	0.049	0.160	0.104	−0.022	2.113	1.732	1.735	1.543
16	985	−0.179	−0.018	−0.089	−0.193	2.029	1.722	1.722	1.535
18	815	−0.173	−0.144	−0.211	−0.123	2.231	2.061	2.061	1.651
20	659	−0.040	−0.100	−0.146	−0.126	2.375	2.177	2.177	1.812
22	372	0.250	0.118	0.114	0.043	2.354	2.073	2.077	1.774
24	264	−0.142	−0.246	−0.191	−0.256	2.415	2.330	2.330	1.930
26	156	−0.169	−0.106	0.013	−0.268	2.483	2.281	2.292	1.824
28	79	−0.356	−0.189	−0.012	0.090	2.444	2.483	2.502	2.031
>30	69	−0.773	−0.573	−0.233	−0.458	2.342	2.611	2.586	2.211

where, β, γ, δ, φ, and θ are considered fixed parameters, common to every plot, u_i a random parameter specific to plot i, h_{ij} and e_{ij} the height and error for the jth observation in the ith plot, respectively, τ is the variance for the random-effect, and other variables are as defined earlier.

Since most of the plots had one measurement, autocorrelation arising from the repeated measurements was ignored. Similarly, some plots only contained a few trees (as low as two). Therefore, within-plot variance–covariance was assumed to be $\sigma_e^2 I$ ($I = n_i \times n_i$ identity matrix). In the case of specifying between-plot variation, there is only one random effects parameter α with the random part u_i. As a result, the variance–covariance matrix of random effects (D in Eq. (7)) becomes $D = \sigma_u^2$.

Eq. (8) was fitted to the model data set using the NLMIXED procedure in SAS. In order to examine the impact of the random parameter on the other parameter estimates, the fit was performed with and without the random parameter, u_i. Estimates for parameters with and without the random-effect, along with fit statistics, are presented in Table 6.

Theoretically, the estimates for fixed parameters by NLMIXED and NLIN procedures should be the same if no random parameter is included in the model. This was true as the parameter estimates using NLMIXED and NLIN procedures were identical for all tree species without the random parameter in the model. The fit statistics, AIC and MSE (σ_e^2), both decreased by including the random parameter (Table 6). The maximum and minimum decrease in the MSEs were for white birch (46%) and red pine (12%), respectively. The variance of the random parameter (σ_u^2) also varied across species from 0.0015 to 0.5081 for red pine and white spruce, respectively.

Other parameter estimates were also affected by the presence of the random parameter. The estimate for the fixed parameter (θ) decreased significantly for species which had relatively higher estimates of variance for the random parameter (balsam fir and white spruce). The change in the fixed parameter estimate for the other species, however, was not pronounced and was not consistent across the species considered. The change in the estimates for the remaining parameters was also fairly minor. Similarly, the magnitude of the change in all parameter estimates was not consistent with the magnitude of the change in the variance of the random parameter across species.

Table 5
Average bias (observed − predicted) and its standard deviation by density (trees/ha) class in predicting total heights of jack pine and black spruce trees using Eqs. (2)–(4) and (6) for validation data sets

Density class	Number of trees	Mean bias				Standard deviation			
		Eq. (2)	Eq. (3)	Eq. (4)	Eq. (6)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (6)
Jack pine									
250	258	−1.075	0.970	0.308	−0.441	3.236	3.035	2.940	1.576
750	1690	0.545	−0.166	−0.389	−0.141	2.788	1.992	2.057	1.448
1250	1707	0.889	−0.055	0.057	0.021	2.913	1.833	1.805	1.273
1750	1411	0.173	−0.306	−0.147	−0.006	2.856	1.880	1.771	1.209
2250	1093	−0.183	−0.204	−0.153	0.009	2.779	1.813	1.665	1.126
2750	1204	−0.589	−0.111	−0.194	0.039	2.465	1.641	1.430	1.061
3250	791	−0.742	0.107	−0.080	−0.003	2.338	1.571	1.326	1.082
3750	398	−0.875	0.019	−0.133	−0.141	2.285	1.572	1.350	1.167
4250	289	−0.856	0.520	0.257	0.007	2.353	1.571	1.352	0.966
4750	209	−0.660	0.673	0.346	−0.131	2.047	1.293	1.042	0.964
5250	183	−0.986	0.377	−0.128	−0.089	1.899	1.393	1.157	0.902
5750	173	−1.204	0.360	−0.137	−0.339	2.312	1.545	1.267	1.232
6250	123	−1.281	0.400	−0.091	−0.180	1.912	1.216	0.931	0.962
6750	66	0.040	1.379	1.003	0.254	2.024	1.337	1.141	0.945
>7000	138	−0.876	0.848	0.091	0.097	1.783	1.197	0.996	0.830
Black spruce									
250	162	0.299	0.581	0.652	−0.103	2.298	2.455	2.467	1.936
750	901	0.005	−0.186	−0.165	−0.379	2.237	2.183	2.175	1.890
1250	1296	0.134	−0.169	−0.169	−0.199	2.178	1.959	1.950	1.781
1750	1270	0.243	−0.018	−0.011	−0.030	1.862	1.680	1.668	1.553
2250	1239	0.009	−0.078	−0.068	0.020	2.054	1.720	1.716	1.436
2750	1065	0.134	0.043	0.057	0.113	1.948	1.565	1.565	1.363
3250	798	−0.248	−0.139	−0.124	−0.049	1.791	1.466	1.476	1.306
3750	713	−0.196	−0.041	−0.021	0.035	1.715	1.390	1.401	1.248
4250	528	−0.262	0.088	0.122	0.021	1.620	1.333	1.345	1.194
4750	225	−0.376	0.013	0.068	−0.207	1.883	1.567	1.580	1.486
5250	140	−0.355	0.028	0.089	−0.019	1.406	1.211	1.223	1.046
5750	147	−0.312	0.106	0.170	−0.074	1.616	1.204	1.212	1.090
6250	80	−0.340	0.353	0.403	−0.024	1.037	0.818	0.807	0.708
6750	63	−0.190	0.221	0.288	0.310	1.690	1.319	1.329	1.113
>7000	108	−0.361	0.088	0.162	−0.177	1.398	1.214	1.217	1.156

Eq. (8) with and without the random parameter was further evaluated using the validation data sets. Heights of the trees in the validation data set were predicted by ignoring the random parameter (fixed-effects response) and using localized parameters (calibrated response). The number of trees measured for both height and diameter for a particular tree species in a given plot was not constant and for some plot and species combinations, it was as low as 1. Therefore, plots with at least two height–diameter observations for the targeted tree species were used in the evaluation. One tree was randomly selected from each plot for each species to predict random-effects. PROC IML in SAS was implemented to predict the random-effect for each plot-species combination using Eq. (7). Calibrated response (predicted height) for each tree in each plot was then obtained using Eq. (8).

Height prediction bias (observed − predicted) was calculated for each fixed and calibrated response for the validation data sets for all tree species. Mean bias and its standard deviation were calculated for each species (Table 7). These values were smaller for calibrated response than for fixed response except for jack pine. In order to further analyze the bias, it was divided into different DBH and density (trees/ha) classes as defined earlier. Average values and corresponding standard deviations were computed by each DBH and density class. Overall, average bias and its standard deviation were slightly smaller for calibrated response than for fixed response for all DBH and density classes for all species.

Fixed and calibrated responses for Eq. (8) were further analyzed by observing residual plots (bias versus predicted height) for red pine and white spruce using the evaluation data set, as the variance of the random parameter for these two species was the smallest and highest, respectively. Residual plots for these tree species were also constructed for Eq. (2) for visual comparisons (Fig. 1). These plots demonstrate that Eq. (8) is superior to Eq. (2) in predicting the heights of these tree species. In the case of fixed and calibrated responses for Eq. (8), calibrated responses seem slightly better than fixed responses in terms of predictive ability, especially for taller trees.

To examine the difference in heights across species, heights were predicted using Eq. (8) without the random-effect parameter for the same value of BA = 25 (m^2/ha), TPH = 2500 (trees/ha), and dominant stand height (SHT) = 20 m for all species (Fig. 2). These plots revealed that the change in

Table 6
Parameter estimates (standard errors in parentheses) and fit statistics for Equation (8) fitted using NLMIXED procedures in SAS for each tree species in the study

Method[a]	θ	δ	β	φ	γ	σ_e^2	σ_u^2	AIC
Balsam fir								
1	6.2496 (0.3427)	0.4024 (0.0177)	0.0650 (0.0035)	0.0604 (0.0099)	1.5017 (0.0410)	2.2006 (0.0526)	–	12675
2	4.7942 (0.3455)	0.4675 (0.0243)	0.0607 (0.0035)	0.0977 (0.0131)	1.5166 0.0389)	1.2964 (0.0362)	0.2920 (0.0450)	11912
Balsam poplar								
1	3.5881 (0.4062)	0.6222 (0.0330)	0.0426 (0.0041)	0.0979 (0.0248)	0.8198 (0.0526)	1.8640 (0.1068)	–	2119
2	3.1718 (0.4441)	0.6508 (0.0429)	0.0397 (0.0043)	0.1482 (0.0334)	0.8763 (0.0594)	1.2783 (0.0881)	0.0357 (0.0120)	2043
Black spruce								
1	3.7096 (0.1245)	0.6206 (0.0103)	0.0451 (0.0016)	0.1239 (0.0060)	1.3222 (0.0243)	2.3205 (0.0304)	–	42659
2	3.3952 (0.1721)	0.6390 (0.0168)	0.0424 (0.0018)	0.1380 (0.0096)	1.2948 (0.0218)	1.4306 (0.0202)	0.0966 (0.0105)	39532
Jack pine								
1	1.1806 (0.0256)	0.9563 (0.0068)	0.0507 (0.0016)	0.2097 (0.0086)	1.0523 (0.0309)	1.4294 (0.0193)	–	34899
2	1.1583 (0.0355)	0.9721 (0.0099)	0.0401 (0.0016)	0.2289 (0.0115)	0.9399 (0.0280)	1.1421 (0.0168)	0.0026 (0.00001)	33986
Red pine								
1	0.9701 (0.0955)	1.0165 (0.0307)	0.0510 (0.0045)	0.2686 (0.0284)	2.1086 (0.2034)	2.6488 (0.1451)	–	2551
2	1.0323 (0.1231)	0.9962 (0.0375)	0.0494 (0.0048)	0.2612 (0.0331)	1.9307 (0.1928)	2.3325 (0.1368)	0.0015 (0.0006)	2526
Trembling aspen								
1	2.0151 (0.0584)	0.7915 (0.0088)	0.0559 (0.0015)	0.1798 (0.0081)	1.1440 (0.0303)	1.9686 (0.0322)	–	26242
2	2.3161 (0.0929)	0.7456 (0.0127)	0.0543 (0.0018)	0.1738 (0.0107)	1.0967 (0.0284)	1.4710 (0.0263)	0.0134 (0.0014)	25328
White birch								
1	2.8705 (0.1450)	0.6426 (0.0164)	0.0497 (0.0026)	0.1680 (0.0122)	1.0539 (0.0310)	2.2516 (0.0473)	–	16541
2	2.3273 (0.1452)	0.6889 (0.0209)	0.0452 (0.0027)	0.2263 (0.0152)	1.1041 (0.0313)	1.2250 (0.0299)	0.0539 (0.0070)	15296
White spruce								
1	9.4916 (0.5878)	0.3717 (0.0175)	0.0342 (0.0022)	0.0310 (0.0102)	1.1071 (0.0269)	2.3877 (0.0632)	–	10592
2	7.4622 (0.6784)	0.4368 (0.0274)	0.0314 (0.0023)	0.0664 (0.0160)	1.1101 (0.0282)	1.4943 (0.0451)	0.5081 (0.0985)	10026

[a] Method 1 = Eq. (8) fitted using NLMIXED procedure without any random parameters and 2 = Eq. (8) fitted using NLMIXED procedure with a random parameter. σ_e^2 and σ_u^2 are error and random-effect variances, respectively.

heights from one DBH level to another is not the same for all species. White spruce has the shortest height followed by balsam fir and black spruce until approximately 22 cm DBH.

The other species did not follow the exact trend but were always taller than the three species mentioned above until approximately 22 cm DBH. Thereafter, they crossed over and white birch became the shortest tree followed by balsam fir when the DBH reached approximately 32 cm. At 40 cm DBH, white spruce overtopped the other species in height development, followed by black spruce.

As a further test of the models performance, fit statistics were compared across the range of species composition, i.e., the model was evaluated for prediction accuracy where a species proportion of the plot total basal area varied from less than 10 percent to 100 percent. There was no observable trend in prediction accuracy across species proportion, and the magnitude of bias was consistent with those observed from the diameter and density class appraisal. Based on this evaluation, it can be concluded that the species-specific height–diameter relationship is independent of the species sampled to calculate dominant stand height. As Calama and Montero (2004) pointed out, predicting the random parameters for a plot using complementary observations of height increased the predictive ability of the model for all tree species studied here.

Table 7
Average bias (observed − predicted) and its standard deviation using fixed and calibrated responses of total heights of all tree species using Eq. (8) for validation data set

Species	Number of trees	Mean bias		Standard deviation	
		Fixed	Calibrated	Fixed	Calibrated
Balsam fir	2907	0.275	0.099	1.478	1.282
Balsam poplar	536	0.434	0.149	1.497	1.338
Blacks spruce	8735	0.210	0.112	1.519	1.346
Jack pine	9733	0.007	0.022	1.224	1.171
Red pine	339	0.104	0.103	1.542	1.470
Trembling aspen	5749	0.051	0.006	1.411	1.274
White birch	3933	0.197	0.106	1.432	1.222
White spruce	2451	0.145	0.062	1.444	1.286

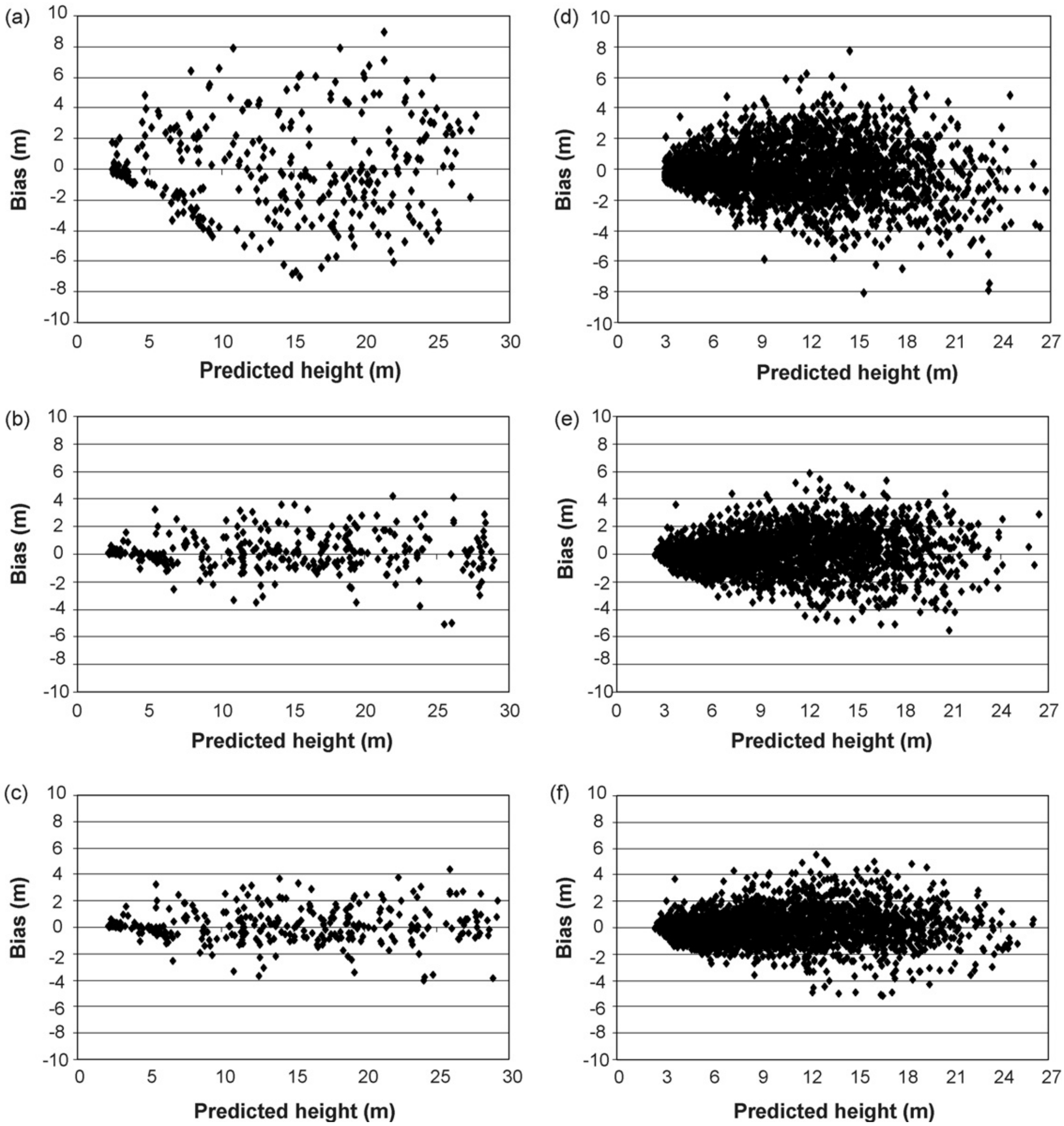

Fig. 1. Bias (observed − predicted) in predicting total heights of red pine; (a) Eq. (2), (b) Eq. (8) fixed responses, and (c) Eq. (8) calibrated responses and white spruce; (d) Eq. (2), (e) Eq. (8) fixed responses, and (f) Eq. (8) calibrated responses.

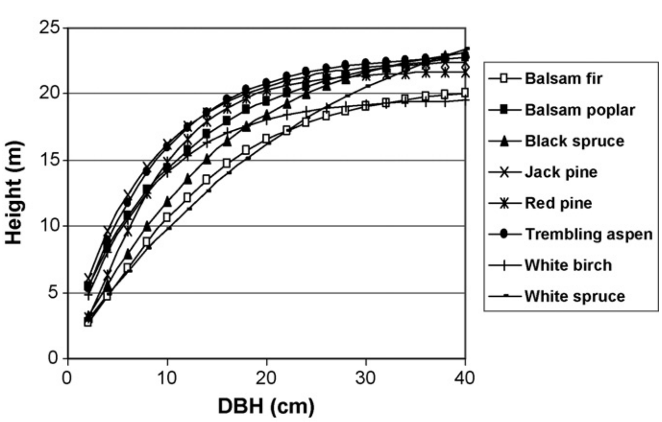

Fig. 2. Predicted tree heights (fixed responses) at BA = 25 m^2/ha, TPH = 2500 trees/ha, and dominant stand height (SHT) = 20 m for all eight tree species using Eq. (8) without the random parameter.

4. Conclusion

The Chapman–Richards function was modified to model the height–diameter relationship of eight major commercial tree species (balsam fir, balsam poplar, black spruce, jack pine, red pine, trembling aspen, white birch, and white spruce) growing in the boreal forests of Ontario, Canada. The asymptote expressed in terms of dominant stand height (average of dominant and codominant heights) and the rate parameter expressed as a function of stand density and basal area, resulted in the best model in terms of fit characteristics (R^2 MSE, and AIC) and prediction accuracy for these tree species. A mixed-effects modeling approach was applied in fitting the models for all tree species.

Fit characteristics (MSE and AIC values) of the mixed-effects model with a random parameter were better than those of its fixed-effects counterpart. Similarly, random parameter predicted using only one randomly selected tree from each plot improved the prediction accuracy of the model.

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