Validation of empirical yield curves for natural-origin stands in boreal Ontario

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ABSTRACT

In Ontario, yield tables for forest management planning have remained relatively unchanged since initial work in the 1950s that was based on a limited number of temporary sample plots. In 2000, the Forestry Research Partnership accelerated work on the Benchmark Yield Curve Project (initiated several years earlier by the Ontario Ministry of Natural Resources, OMNR) to update these tables. The resulting yield curves incorporated data from more than 3000 permanent sample plots (PSPs) maintained in Ontario as well as PSPs from neighbouring and ecologically similar jurisdictions. Two stratifications were considered: OMNR's Northeast Region standard forest units and leading species. The 10 forest units considered cover the major commercial species in the boreal forest in Ontario. Equations were fit to the data to predict the growth and yield by stratum. The equations were validated against independently collected data and compared to predictions from the current wood supply yield curves in Ontario: Plonski's yield tables, modified Plonski, and northeast regional curves. Results of the validation showed that, with the exception of the MW2 and SF1 forest units, the new yield curves generally had less bias for gross total volume than Plonski and modified Plonski. Results for net merchantable volume were consistent with those for gross merchantable volume. The MW2 and SF1 forest units are more mixed in terms of species type, species light tolerance, and age. A leading species approach resulted in better predictions and is recommended for these forest units.

Key words: wood supply, benchmark yield curves, mixedwood yield, yield model, Forestry Research Partnership

RÉSUMÉ

Les tables de rendement utilisées en Ontario à des fins de planification de l'aménagement forestier n'ont guère changé depuis les travaux initiaux entrepris dans les années 1950 et reposant sur un nombre restreint de places-échantillons temporaires. En 2000, le Partenariat pour la recherche forestière a accéléré les travaux de mise à jour des tables entrepris par le Projet de courbes de rendement de référence (amorcés quelques années auparavant par le ministère des Richesses naturelles de l'Ontario, MRNO). Les courbes de rendement obtenues incorporaient des données tirées de plus de 3 000 parcelles échantillons permanentes (PEP) mises en place en Ontario ainsi que de PEP de juridictions avoisinantes et écologiquement similaires. Deux stratifications ont fait l'objet d'études : les unités forestières standards de la Région du nord-est du MRNO et les espèces dominantes. Les dix unités forestières sous étude regroupaient les principales espèces commerciales de la forêt boréale de l'Ontario. Les équations ont été adaptées aux données afin de prédire la croissance et le rendement pour chaque strate. Les équations ont été validées par rapport à des données recueillies de façon indépendante et comparées aux prédictions générées par les courbes actuelles de rendement de matière ligneuse de l'Ontario, les tables de rendement de Plonski, les tables modifiées de Plonski et les courbes régionales du nord-est. Les résultats de la validation ont démontré que, exception faite des unités forestières MW2 et SF1, les nouvelles courbes de rendement présentaient généralement un biais plus faible au niveau du volume total brut que dans le cas des courbes de Plonski et des courbes modifiées de Plonski. Les résultats dans le cas du volume marchand net étaient constants avec ceux du volume marchand brut. Les unités forestières MW2 et SF1 présentent un mélange plus accentué en terme de types d'espèces, d'espèces tolérantes et d'âge. L'approche en fonction de l'espèce dominante a généré de meilleures prédictions et est recommandée pour ces unités forestières.

Mots clés : approvisionnement en bois, courbes de rendement de référence, rendement des forêts mélangées, modèle de rendement, Partenariat pour la recherche forestière.

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Introduction

Yield tables in Ontario have changed little since initial work in the 1950s from which hand-drawn curves were developed based on a limited number of temporary sample plots. Since then, requests for the development of new growth and yield models have been ongoing.

In 1991, ESSA Technologies Ltd. was contracted to review the Ontario Ministry of Natural Resources (OMNR) Growth and Yield Program to recommend a program plan (Kurz *et al.* 1991). Much of the focus was on augmenting the existing network of permanent sample plots (PSPs). The recommendations led to revised field methods (Hayden *et al.* 1995) and the establishment and maintenance of thousands of PSPs by the OMNR and the Forest Ecosystem Science Cooperative Inc (FESC) Growth and Yield Science Unit[5]. The need for growth and yield models to project forest growth in managed and unmanaged forests of Ontario was noted in the program plan (Kurz *et al.* 1991).

During an intensive forest management workshop in Sault Ste. Marie in 1999, participants identified the following growth and yield-related needs (Bell *et al.* 2000, p. 31):

- Locally calibrated (region-, subregion-, and forest management unit-specific) yield curves for the full spectrum of silvicultural treatment options based on forest units and ecosites;
- Yield curves for mixedwood stands;
- Yield curves for managed stands including plantations established with improved stock and/or subjected to vegetation management; and
- Yield curves for partial harvesting and thinning regimes

Participants also identified the need for objective peer-reviewed programs and projects to ensure reliability of the information.

The Ontario Forest Accord (OMNR 1999) outlined 31 commitments agreed to by members of the forest industry, the Partnership for Public Lands, and OMNR. Commitment 5 called for the development of an Ontario forest science partnership, in part to assess the effects of intensive forest management on increased forest growth and yield. Tembec Industries' response to this commitment led to the creation of the Forestry Research Partnership (FRP) among Tembec, OMNR, and the Canadian Forest Service.

Yield curves are an integral part of forest management planning. The FRP, as part of a strategic initiative to increase wood supply on a reduced landbase (Bruemmer 2008, this issue), undertook a project to accelerate the development of yield curves based on field data. These curves were required to represent a range of current and potential management intensities for stratification based on the standard northeast forest units (Watt *et al.* 2001) and overlapping south central (draft) forest units. The curves were required to be compatible with the strategic forest management model (SFMM) (Davis 1999) and applicable to forest management units (FMUs) in Ontario. The intent of the project was to use all available permanent sample data of known origin, relevant to Ontario.

The objectives of this study were to:

- develop empirical yield curves for use in forest management planning,
- test these curves against independent data,
- compare the forest unit and leading species approaches to yield curve development, and
- compare the precision and accuracy of the new yield curves against currently available yield prediction tools in Ontario.

[5]http://www.forestco-op.ca/projects_gysu/pgp.htm

Existing yield curves

In 1956, W.L. Plonski published normal yield tables for black spruce (*Picea mariana* (Mill.) BSP), jack pine (*Pinus banksiana* Lamb.), aspen (*Populus tremuloides* Michx.) and white birch (*Betula papyrifera* Marsh.) in Northern Ontario based on temporary sample plots (see Plonski 1956). The original yield tables were later expanded to include tolerant hardwood and white pine (*Pinus strobus* L.) and red pine (*Pinus resinosa* Ait.) stands in Ontario (Plonski 1960) followed by a metric version (Plonski 1981). Plonski's (1981) tables were based on fewer than 900 temporary sample plots with the bulk of the data collected prior to 1960 (see Table 1). The number of species represented is limited and the yield curves generally end at around age 100 for intolerant species. Since the 1960s, Plonski's yield tables, or a variant, have been used for forest management planning on public lands in Ontario without a significant addition of data and have undergone several, undocumented modifications.

Titus and Morton (1985) predicted that the increased power of computers and complexity of forest management planning would lead to increased use of growth and yield models. However, Ontario has lagged behind other jurisdictions such as Quebec (Pothier and Savard 1998), Alberta (Huang *et al.* 2001) and British Columbia (Martin 1991, Garcia 2001) in developing such models. Payandeh (1991) fit equations to Plonski's tables to allow for interpolation and extrapolation. He fit the various attributes as functions of site index and age. However, unlike Payandeh's formulation, Plonski's yield tables are polymorphic (particularly black spruce basal area) so the yield tables remain the definitive source for Plonski's estimates.

SFMM (Davis 1999) is a corporate provincial software application used in evaluating different management alternatives and scenarios in Ontario. The model's preprocessor, SFMMTool (Watkins 2004), takes the forest resource inven-

Table 1. Number of temporary sample plots used by Plonski (1981) to develop the normal yield tables. Numbers are given by species as well as for the number of trees destructively sampled to develop volume equations

Species	Number of plots	Number of trees sampled for volume
Black spruce	224	1902
Spruce (site class 1a)	70 (2178[a])	
Jack pine	181	1336
Aspen	82	1132
White birch	59	587
White Pine	90	
Red pine	165	
Tolerant hardwood	NA	
Total	**871**	**4957**

[a]2178 Forest Resource Inventory sample plots were used.

tory (FRI), stratifies it into forest units, and assigns yield curves. The default yield curves available in SFMMTool are Plonski, modified Plonski, and northeast regional yield tables derived from Plonski's (1981) tables.

The modified Plonski predictions available in SFMMTool (Watkins 2004) have little documentation. Essentially, modified Plonski adds net merchantable volume (NMV – the gross merchantable volume [GMV] minus volume lost to cull) and extends the original curves to age 250, generally by predicting a decrease in NMV to zero by approximately age 160. This version shows a much slower drop in volume of white and black spruce but volumes for tolerant hardwoods, hemlock (*Tsuga canadensis* (L.) Carr.), and white and red pine are constant past 150 years. Gross total volume (GTV), NMV, current annual GTV increment, and mean annual GTV increment are available for modified Plonski in SFMMTool.

Northeast regional curves (developed by Neil Maurer, formerly with OMNR) are also based on temporary sample plots, primarily in natural, untreated stands. These curves are generally similar to modified Plonski with a sharper decline with age in net merchantable volume for jack pine, white spruce, poplar, and white birch. For the northeast regional curves, only net merchantable volume is available in SFMMTool. Rationale for the decline in NMV with age is not documented. SFMMTool includes pure species yield tables by site and age class and estimates the yields of mixed species stands as the sum of yields from pure species stands, weighted by the proportion of the species composition. The FRI attribute stocking scales the volume. Stocking is measured in the field as the actual basal area relative to the basal area of a fully stocked stand with the same leading species and site class as given in Plonski (1981). SFMMTool assumes a stand with 80% stocking has 80% of the volume of a fully stocked stand of the same age, site class, and species composition.

To date, the most common yield prediction models used in forest management planning in Ontario are Plonski, modified Plonski, and northeast regional yield tables. Yield curves developed by Pothier and Savard (1998) for Quebec were also tested and are referred to here as Pothier predictions.

Forest units

Forest management planning has gone from a stratification based on leading species (the species or species group with the most basal area in a stand) to one based on forest units. A forest unit is defined as *an aggregation of forest stands for management purposes which have similar species composition, develop in a similar manner (both naturally and in response to silvicultural treatments), and are managed under the same silvicultural system* (OMNR 2004). The northeast standard forest units (Watt *et al.* 2001) are presented in Table 2. Note that not all forest units have a single dominant species. In the Romeo Malette Forest, for example, the average species composition for the MW1 forest unit is 34% jack pine, 22% poplar, 19% white birch, 11% black spruce, 2% white spruce, and 2% balsam fir (*Abies balsamea* [L.] Mill.). If the total area of the MW1 is broken down by leading species, approximately 40% has jack pine as a leading species, 34% poplar, and 20% white birch with small areas in balsam fir, cedar, and black spruce. Generally, the leading species of a stand within the mixedwood forest unit comprises at least 40% of the species composition.

Succession

In forest management planning in Ontario, non-stand replacing succession is not incorporated directly into yield curves. Succession transition matrices are created that predict, for each age class, the proportion of a forest unit that moves to a different forest unit and its age. Thus, succession is separate from yield. Although SFMMTool predicts yield curves to age 255, the succession rules move all the area to younger age classes well before the trees reach that age.

Table 2. Summary of the northeast standard forest units (taken from Watt et al. 2001)

Forest Unit	Description
BW1	Stands are hardwood dominated by white birch. They occupy some of the same sites that PO1 stands occupy as well as somewhat drier and coarser textured soils.
LC1	Stands are mixtures of black spruce, larch and/or cedar occupying wet, moderately deep organic soils associated with drainage ways or the toe of slopes where telluric water augments the on-site nutrient pool.
LH1	Stands are meant to capture rich low lying areas with black ash, balsam poplar and American elm and red maple.
MW1	Stands are mixed coniferous–deciduous comprising trembling aspen, white birch, jack pine and black and white spruce. They occur on dry to moist sandy to coarse loamy soils.
MW2	Stands are mixed coniferous–deciduous comprising mostly trembling aspen, white birch, black and white spruce and balsam fir. They occupy fresh to moist, medium loamy to clayey soils.
PJ1	Stands are nearly pure jack pine growing on dry to fresh, sandy to coarse loamy soils of glaciofluvial origin.
PJ2	Stands are coniferous with jack pine and black spruce growing on dry to moist sandy to coarse loamy soils of glaciofluvial origin.
PO1	Stands are hardwood dominated by trembling aspen. They typically occur on fresh to moist loamy to clayey soils with free carbonates present in the upper 100 cm.
SB1	Stands comprise nearly pure black spruce growing on wet deep organic soils and on moist peaty-phased mineral soils in lower slope positions.
SF1	Stands are mixed conifer with white spruce, balsam fir, black spruce, and eastern white cedar growing on moist sandy to clayey soils. They are often found on lower slope positions associated with telluric seepage.
SP1	Stands are upland black spruce dominated conifer on fresh to moist medium loamy to clayey soils.

Methods

The modelling approach used here was to predict stand-level attributes that included basal area per hectare ($m^2 \cdot ha^{-1}$), density ($stems \cdot ha^{-1}$), top height (m), gross total volume (GTV, $m^3 \cdot ha^{-1}$), and net merchantable volume (NMV, $m^3 \cdot ha^{-1}$) from stand-level FRI attributes. Whole-stand models have been used extensively for pure species, even-aged stands, but may be less useful for mixed-species stands that potentially contain multiple age and size classes (Vanclay 1994). The process of determining an appropriate model structure starts with determining the needs of the user—the questions to be addressed, the required accuracy of estimates, and the range of conditions to which the model will be applied (Battaglia and Sands 1998).

A stand-level modelling approach was used for 2 main reasons:

- The intended application of the model was in forest management planning. In Ontario, the primary stand-level attribute in forest management planning is NMV by species.
- The inputs used with the model were stand-level data, in particular, attributes available from the FRI. The FRI includes species composition, stand age, height, and stocking (a measure of stand basal area).

It was anticipated that developing curves from repeated measurements on permanent sample plots (PSPs) and using all the plot data available in Ontario would provide improved yield estimates and, most importantly, reliable empirical estimates of growth.

Empirical models continue to serve an important function in predicting the yield of wood fibre (Korzukhin *et al.* 1996). The models developed here are not intended to address all growth and yield concerns in Ontario. In particular, mixed species conditions and mid-rotation density regulation (thinning, partial harvesting) will require different modelling approaches. One such model being adapted for use in Ontario is FVS^{Ontario} (www.fvsontario.ca) (Lacerte *et al.* 2006). FVS^{Ontario} is the Ontario variant of the Forest Vegetation Simulator (FVS), the official growth model of the United States Forest Service. FVS is a distance-independent, individual-tree model. It requires a tree list to initialize predictions. Much work has gone into developing tree list generation models to link FVS to stand-level (inventory) attributes. It is anticipated that predictions from the present yield curves and those from FVS^{Ontario} will be similar for even-aged, relatively pure species conditions. However, FVS^{Ontario} will likely give superior predictions for the growth of mixed species stands since trees are grown individually.

To be useful in wood supply modelling, the yield curves had to be compatible with the FRI, which consists of species composition (to the nearest 10% for each species), age class, stocking, average height of dominant and codominant trees, and site class). Various other classification variables were also available, including ownership, site region, and site district. Generally, silvicultural history is not available as part of an interpreted FRI. However, recent efforts have seen past silvicultural investments and treatments incorporated spatially into new inventories.

According to Vanclay (1994), yield equations assume a prescribed set of management activities. Growth equations in a whole stand model context have the advantage that broad silvicultural treatments, such as thinning and spacing, can be simulated at any time. Yield equations should generally be appropriate for natural and extensive conditions with no management intervention. They may also be used to predict growth under unmanaged conditions assuming that stocking is constant over time.

The potential independent variables were attributes available from the inventory and included age, site index, stocking, forest unit, and stand origin. The yield equations were fit by forest unit.

Data

The sample plots from which data were obtained were distributed across the productive forest area in Ontario (Fig. 1) and encompassed a range of conditions (Table 3). Data were from the Ontario provincial database as well as the FESC Growth and Yield Science Unit, the Canadian Forest Service, and the Quebec Ministère des Ressources naturelles et Faune. Ontario's historic as well as new PSPs and Permanent Growth Plots[6] (PGPs) established by the OMNR were used for calibration data; validation data came from the PGPs of the FESC. Most of the data were from fixed area plots with individually tagged trees but a small proportion was from variable radius plots. In a number of the older data sets, the trees were not tagged but were tallied by diameter class. Those that have been remeasured were upgraded to the current standard.

Stratification

The PSPs were stratified by standard forest units (Table 2), with hemlock included in the tolerant hardwood forest unit. Because not all forest management plans use the standard forest units and some forest units have more heterogeneous species compositions, 2 alternate stratifications were examined. To address the non-standard forest units, the approach of stratifying plots by leading species was tested. To address the mixed species forest units, the MW1 and MW2 were further stratified by leading species within forest unit.

Data representativeness

Data analysis focused on ensuring the model forms fit the data. It was essential that the data represent the full range of conditions within the population. However, it was not essential that the plots be a random sample from the population (Iles 2003) or be representative or numerically proportional to forest type area (Vanclay 1994). In general, historic plots such as those of the AmericanCan and Kimberly Clark systems were located on better sites with high stocking and represented better-than-average conditions. Experimental sites such as the Thunder Bay spacing trial (McClain *et al.* 1994) and those from the Petawawa Research Forest (e.g., Burgess and Robinson 1998) generally had above-average growth due to complete site occupancy, above-average protection, and better tending. To avoid bias these attributes were included as covariates in the model. The more recently established growth and yield plots (1994 and ongoing) are more representative of the range of conditions in the population.

[6]PSPs and PGPs are closely related plot designs. A PGP consists of a single 400-m^2 circular plot in which all trees with diameter at breast height ≥2.5 cm are tagged and evaluated. A PSP consists of a cluster of 3 PGPs nested within a larger 6400-m^2 circular plot that is used to assess tree mortality and snag dynamics.

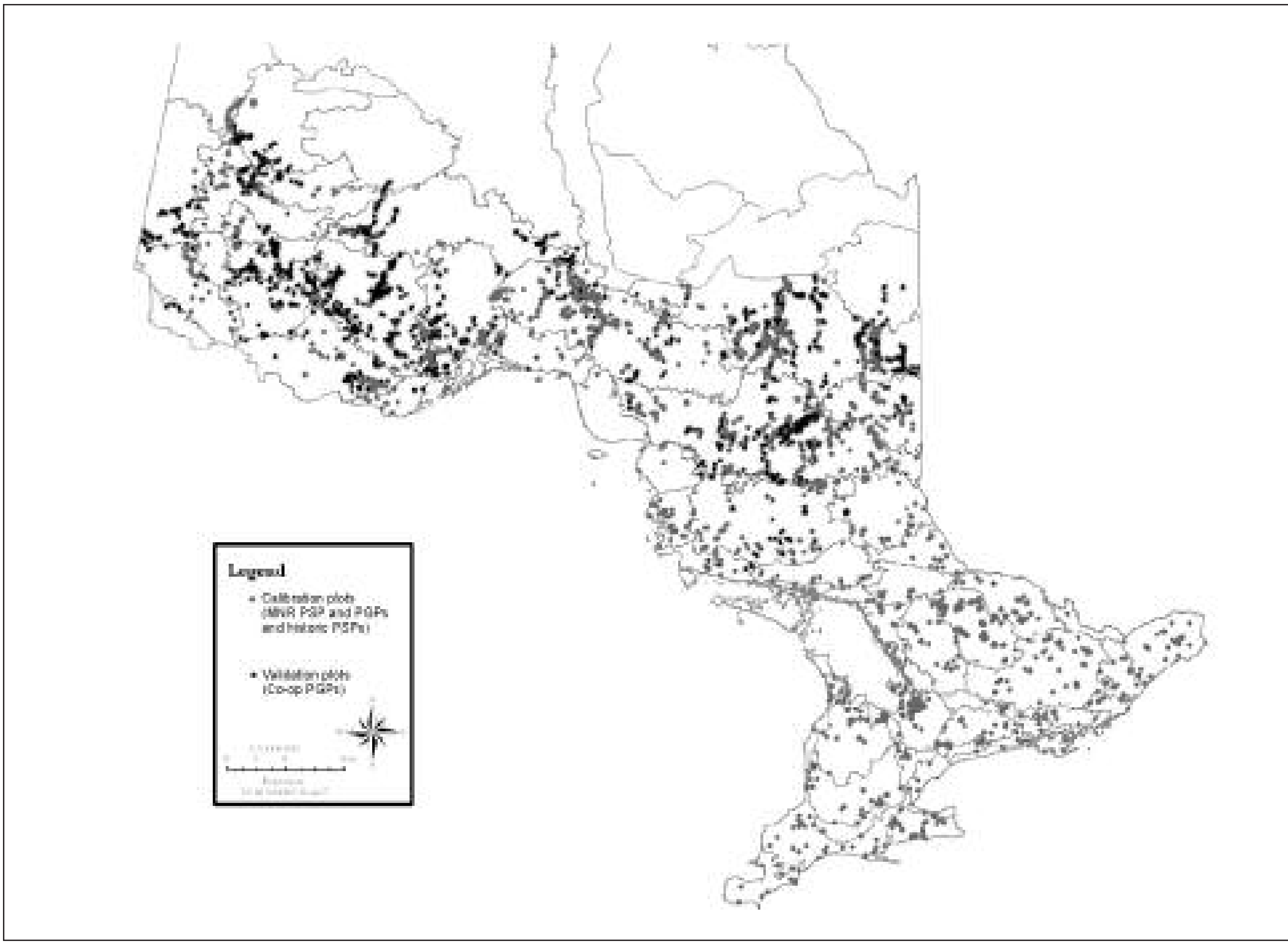

Fig. 1. Locations of the historic permanent sample plots (PSPs) and Ontario Ministry of Natural Resources PSPs and permanent growth plots (PGPs) (calibration data) along with the Forest Science Co-op PGPs (source of validation data).

The Quebec plots use a nested design (MRNQ 2001). Trees with diameter at breast height (DBH) >9 cm are measured on a 0.04-ha plot and trees with DBH between 1 cm and 9 cm are tallied by 2-cm DBH class on a 0.004-ha plot. The basal area and volume estimates are relatively precise but the density (stems·ha^{-1}) estimates have high variability. The Quebec plots were not used to estimate the density functions for aspen, black and white spruce, and white birch.

Data compilation

Data compilation and analysis were conducted using SAS® BASE and STAT statistical software. The graphs were produced using Microsoft Excel®.

Stand age was determined using 1 of 2 methods. If the year of stand origin was available, stand age was calculated as the difference between the measurement year and the year of stand origin. For the remaining stands, the leading species was defined as the species with the greatest basal area. The average total age (weighted by basal area) of increment-cored trees of the leading species was assumed to be the stand age. If ages from more than a single measurement were available, the average stand age calculated from the various measurements was used to determine the year of origin. If total age was not available but age at breast height was, the average years to breast height was added to the breast height age to estimate total age. The average years to breast height was calculated by species for those trees with total and breast height age measurements. Most boreal plots were established in even-aged stands that established following stand-replacing disturbance. Thus, the number of years to reach breast height may be considerably underestimated for late-successional plots dominated by species such as black spruce and balsam fir that may have existed for many years in the understory before becoming dominant.

Missing heights were estimated by fitting height (Ht)–DBH curves to the data using the following variation of the Bertalanffy equation (Pienaar and Turnbull 1973).

$$[1] \qquad \hat{ht} = 1.3 + (a_0 + a_1 \cdot age) \cdot (1 - e^{b_0 \cdot DBH^{b_1}})$$

Sample trees included those measured for height and diameter in the growth plots and those measured for increment outside the growth plot. The minimum sample size for fitting a separate ht–DBH curve was 10 observations. If 10 or more observations were available for a particular species, plot, and remeasurement combination, these were used to estimate a curve for that combination (in this case, the term including *age* was dropped from the equation). Curves were fit by species × ecoregion and by species For plots with no age information, a species-level curve was used (and the *age* term omitted).

Table 3. Basic growth and yield sample plot statistics by forest unit. Means are followed by the data ranges in parentheses.

Forest Unit	Number of plots	Number of measurements	Age (years)	Basal area ($m^2 \cdot ha^{-1}$)	Density ($stems \cdot ha^{-1}$)	Stocking[a]	Site index (m)	Gross total volume ($m^3 \cdot ha^{-1}$)
Calibration data								
BW1	1141	1725	60 (20–148)	19 (0–50)	1367 (25–6650)	0.94 (0–2.56)	15.6 (6–36)	119 (0–344)
LC1	326	468	82 (21–192)	26 (0–84)	1737 (25–5300)	0.67 (0.01–1.75)	12.7 (2–38)	140 (0–509)
LH1	1233	2321	58 (20–163)	21 (0–54)	1310 (25–5000)	0.81 (0.02–1.94)	18.3 (6–34)	129 (2–413)
MW1	175	303	67 (21–168)	27 (1–48)	1654 (125–5547)	1.05 (0.05–1.82)	17.2 (9–31)	200 (6–432)
MW2	1792	2752	63 (21–214)	23.3 (1–69)	1607 (50–9375)	0.85 (0.02–7.75)	16.8 (7–37)	147 (3–540)
PJ1	931	2306	55 (20–152)	23.3 (0–49)	2049 (25–17075)	1 (0–1.99)	15.5 (10–30)	163 (0–457)
PJ2	328	492	73 (20–176)	25.1 (1–47)	1639 (50–6009)	1.01 (0.05–2.02)	15.6 (9–31)	179 (3–408)
PO1	1018	1705	60 (20–176)	28 (0–68)	1455 (20–6400)	1.02 (0–3.61)	20.1 (10–35)	223 (0–685)
SB1	2558	4888	98 (20–244)	18 (0–57)	1729 (25–8600)	0.5 (0–2.18)	10.1 (0–32)	97 (0–393)
SF1	3630	4321	72 (20–216)	24 (0–75)	1986 (25–10900)	0.64 (0–2.53)	14.7 (2–38)	129 (0–402)
SP1	804	1333	74 (21–211)	23.1 (0–54)	1771 (75–6125)	0.61 (0.02–1.67)	13.6 (4–31)	145 (2–399)
Validation data								
BW1	57	59	59 (23–107)	21.5 (9–38)	1444 (450–3075)	1.02 (0.52–1.81)	17.1 (10–29)	144 (41–316)
LC1	24	27	96 (48–198)	25.1 (10–49)	2162 (1083–4275)	0.69 (0.31–1.42)	10.7 (5–18)	138 (54–310)
LH1	3	3	58 (46–73)	19.9 (13–25)	1186 (825–1683)	0.84 (0.67–1.02)	17.1 (16–19)	130 (79–173)
MW1	61	62	53 (20–120)	22.4 (8–35)	1744 (367–4175)	0.93 (0.3–1.63)	19 (11–26)	155 (44–317)
MW2	146	152	63 (21–144)	23.8 (7–48)	1630 (250–4825)	0.87 (0.33–1.85)	17.4 (10–29)	161 (35–338)
PJ1	258	282	56 (20–118)	23.4 (5–42)	1723 (325–5400)	0.91 (0.33–1.61)	18 (10–26)	166 (19–339)
PJ2	87	91	62 (20–141)	25 (8–47)	1843 (375–3975)	0.98 (0.33–1.81)	17.9 (11–25)	174 (36–392)
PO1	159	171	57 (20–122)	24.4 (5–47)	1506 (225–4817)	0.91 (0.33–1.58)	21.3 (12–30)	195 (22–408)
SB1	202	220	89 (23–193)	24.5 (9–52)	2186 (575–5600)	0.65 (0.3–1.29)	12.5 (5–23)	142 (33–350)
SF1	137	144	59 (21–138)	23.4 (6–47)	1959 (275–5050)	0.61 (0.31–1.11)	15.3 (7–25)	136 (18–309)
SP1	128	132	70 (22–151)	23.9 (7–44)	1874 (317–5375)	0.64 (0.3–1.36)	14.2 (7–24)	152 (25–299)

[a]Stocking is the actual basal area relative to the basal area of a fully stocked stand with the same leading species and site class as in Plonski (1981).

Volumes were estimated using Zakrzewski's (1999) taper model fit to additional species. Merchantable volumes were calculated using the minimum standards in the scaling manual (OMNR 1995)[7], which are a stump height of 30 cm and a minimum top diameter of 16 cm (white and red pine, hemlock, poplar, or white birch), 10 cm (other conifer), or 20 cm (other hardwood).

Analytical assumptions

The tree- and plot-level observations in this study were not independent. Some plots had as many as 8 measurements and some stands had more than 1 plot. Repeated measurements on the same plots tend to be correlated as are measurements from plots that are in close proximity. When ordinary least squares techniques are used with such data, the parameter estimates are unbiased, but the covariance matrix associated with the parameter estimates and the equation variances may be underestimated (Vanclay 1994). No effort was made here to account for the correlations between errors. The simplicity of the analysis and the unbiased parameter estimates were considered to more than compensate for the underestimation of variance.

[7]Ontario's scaling manual was revised in April 2007; results here are based on the previous version (OMNR 1995).

Models

The following models were calibrated from the plot data. They were fit by forest unit, leading species, and, for the MW1 and MW2 forest units, by leading species within forest unit.

Basal area

For even-aged forests, basal area increases with site index and stocking. Basal area also increases with age with a rapid increase at young ages that slows as the stand achieves full site occupancy. Basal area was predicted as a linear function of stocking and site class and a sigmoidal function of age using the following equation form:

$$\hat{BA} = stocking \cdot (\alpha_0 + \alpha_1 \cdot \hat{SI}) \cdot (1 - e^{-\beta_0 \cdot age^{\beta_1}}) \quad [2]$$

The *stocking* $\cdot (\alpha_0 + \alpha_1 \cdot \hat{SI})$ term represents the upper asymptote of basal area, the maximum basal area that a stand with that stocking and site index (SI) can achieve. The remainder of the equation predicts how rapidly the basal area approaches that maximum.

Eq. [2] should be a relatively good predictor of basal area since stocking is the ratio between actual and theoretical (Plonski) basal area. In the FRI, stocking is generally estimated from aerial photography, not calculated, and the accuracy and precision of that estimate is unknown.

Table 4. Parameters and errors associated with the basal area model ($\hat{BA} = stocking \cdot (\alpha_0 + \alpha_1 \cdot \hat{SI}) \cdot (1 - e^{-\beta_0 \cdot age^{c_0}})$) by forest unit and leading species.

Stratification	a0	a1	b0	c0	Mean squared error	Mean error as % of mean
Forest Unit						
BW1	4.1263	1.0792	0.0010	2.0009	3.747	0.6%
LC1	34.0036	0.5398	0.0010	1.9269	9.6220	-0.2%
LH1	10.3683	1.1937	0.0039	1.5178	21.8690	2.6%
MW1	19.0188	0.5225	0.0247	1.1444	17.6700	1.3%
MW2	13.9534	0.6373	0.0010	2.1459	104.4940	11.5%
PJ1	7.9943	1.0715	0.0508	1.0473	1.6050	0.3%
PJ2	21.1641	0.2821	0.1918	0.7411	12.6770	0.9%
PO1	10.3024	1.0943	0.0042	1.5642	3.0680	0.5%
SB1	22.5524	1.5078	0.0021	1.6995	2.5080	0.7%
SF1	38.8072	0.0000	0.0143	1.3558	11.7870	0.4%
SP1	33.4669	0.3725	0.0011	2.0775	13.2210	1.1%
Leading species						
Jack pine	8.8658	1.0142	0.0495	1.0605	1.6680	0.2%
White spruce	20.9329	1.9172	0.0107	1.1911	3.1990	0.7%
Black spruce	23.6583	1.3499	0.0010	1.9412	2.7340	0.4%
Balsam fir	30.4029	0.7456	0.0010	2.0115	4.0940	0.4%
Cedar	40.0000	0.0000	0.0298	1.1316	19.6000	3.5%
Larch	26.4787	1.1630	0.0010	1.9230	2.7220	0.2%
White birch	7.8047	0.8069	0.0010	2.0476	19.4000	3.7%
Trembling aspen	4.2716	1.3007	0.0019	1.8070	18.9540	2.0%
Leading species within Forest Unit						
MW1 – Aspen	15.4013	0.8857	0.0118	1.2716	0.2820	-0.1%
MW1 – Birch	10.0000	0.9024	0.0099	1.3424	0.5336	-0.5%
MW1 – Pine	12.1807	0.8216	0.0608	1.0296	1.8121	0.1%
MW1 – Spruce	23.5063	2.0164	0.0756	0.7302	1.0325	0.0%
MW2 – Aspen	12.6951	1.0069	0.0053	1.4879	0.3109	0.1%
MW2 – Birch	15.3532	0.3884	0.0010	2.0788	2.6864	0.4%
MW2 – Spruce	31.0165	0.7657	0.0010	1.9991	23.0050	0.6%

Top height and site index

Site index curves are an essential part of the yield curves developed here. Historically, height data have been expensive to collect and highly variable. As well, they are generally lacking for minor species. As a result, site index curves from the literature were evaluated against the observed height development patterns in the data. Woods and Miller (1996) was used for white and red pine; Carmean *et al*. (2006) for trembling aspen and black spruce; Carmean *et al*. (2001) for jack pine; Carmean (1996) for white spruce, balsam fir, white birch, and tamarack (*Larix laricina* [Du Roi] K. Koch); and Carmean *et al*. (1989) for the remaining species. Some site index curves use breast height age rather than total age. The age to reach breast height was assumed to be 6 years for all species.

Density

Stand density (stems·ha^{-1}) changes with stocking, age, site index, and basal area. Several variations in model form were tried, including raising the independent variables to negative exponents and using the inverse of the independent variables. When comparing the alternative equations, particular attention was paid to the predictions at older ages.

The following equation form provided good predictions for density (stems·ha^{-1}).

[3] $$density = \frac{\gamma_0}{SI} \cdot stocking^{-\gamma_1} \cdot \hat{BA} \cdot age^{-\gamma_2}$$

The variance of the residuals increased with predicted density so the observations were weighted by $\frac{1}{\hat{BA}}$, resulting in a more homogenous variance of the residuals. The density and BA predictions are used to estimate the quadratic mean DBH, which is required for some of the volume predictions.

Volume

The following equation form was used to predict gross total stem volume (GTV) (m^3 ha^{-1}). The coefficient is analogous to the cylindrical form factor $f = \frac{vol}{BA \cdot ht}$ (Husch *et al*. 1972).

[4] $$\hat{GTV} = z_0 \cdot \hat{BA} \cdot \hat{topht}$$

The variance of the residuals increased with predicted volume so the observations were weighted by $\frac{1}{\hat{BA}}$, resulting in a more homogenous variance of the residuals.

Table 5. Parameters and errors associated with the density model ($density = \frac{x_0}{SI} \cdot stocking^{-x_1} \cdot \hat{BA} \cdot age^{-x_2}$) by forest unit and leading species

Stratification	x0	x1	x2	Mean squared error	Mean error as % of mean
Forest Unit					
BW1	855258	0.0373	1.6118	205536	-4.6%
LC1	93928	0.3970	1.1296	790393	-2.0%
LH1	102395	0.4008	1.1511	440283	-2.0%
MW1	181874	0.0848	1.2542	488365	0.5%
MW2	86920	0.2855	1.0416	649810	-0.7%
PJ1	444677	0.1335	1.4972	959669	0.9%
PJ2	39192	0.0000	0.8634	596174	0.1%
PO1	395470	0.0000	1.4526	320293	-3.7%
SB1	150955	0.1559	1.1321	669925	-4.7%
SF1	29524	0.0000	0.8067	1035076	-6.9%
SP1	106842	0.0213	1.0964	389494	0.4%
Leading species					
Jack pine	371769	0.1103	1.4448	924466	1.5%
White spruce	123533	0.0292	1.1045	1420945	-9.1%
Black spruce	98109	0.0551	1.0630	532049	-3.4%
Balsam fir	203256	0.0232	1.2901	633577	-1.0%
Cedar	145068	0.0426	1.2451	223222	-0.8%
Larch	476742	0.1540	1.4922	44001	-0.7%
White birch	135048	0.0003	1.1697	386455	-2.0%
Trembling aspen	298072	0.0632	1.3720	417467	-1.3%
Leading species within Forest Unit					
MW1 – Aspen	225674	0.0000	1.2918	624424	0.0%
MW1 – Birch	9966	0.0000	0.5095	980784	0.0%
MW1 – Pine	114553	0.0000	1.1665	179404	0.0%
MW1 – Spruce	184284	0.0000	1.2686	436478	-4.6%
MW2 – Aspen	121470	0.0044	1.1522	386295	0.0%
MW2 – Birch	159919	0.0000	1.2275	402306	4.0%
MW2 – Spruce	229392	0.0059	1.3065	644666	0.0%

Gross merchantable stem volume (GMV) was predicted as a proportion of the total volume where the proportion is a function of quadratic mean DBH (Dbh_q).

[5] $$\hat{GMV} = z_3 \cdot (1 - e^{-z_4 \cdot Dbh_q^{z_5}}) \cdot \hat{GTV}$$

Net merchantable volume (NMV) was predicted from GMV by subtracting an estimate of cull. The following equation was fit to the cull factors from OMNR (1978), by species. Equation (6) predicts the cull fraction, which increases as a sigmoidal function of age.

[6] $$\hat{cull}_i = (1 - e^{-d_1 \cdot age})^{d_2}$$

For each species, the ratio of GTV to basal area (VBAR) was estimated as a function of age using the entire data set.

For each species, the net merchantable volume was estimated using the following equation.

Table 6. Parameters and errors associated with the gross total volume model ($\hat{GTV} = z_0 \cdot \hat{BA} \cdot \hat{topht}$) by forest unit and leading species

Stratification	z0	Mean squared error	Mean error as % of mean
Forest Unit			
BW1	0.3696	347.7	1.1%
LC1	0.3566	433.1	-1.4%
LH1	0.3524	1597.7	9.2%
MW1	0.3734	1208.4	3.5%
MW2	0.3200	5671.4	19.7%
PJ1	0.4209	549.4	4.6%
PJ2	0.3838	1286.5	3.7%
PO1	0.3536	621.5	1.2%
SB1	0.3707	461.9	0.0%
SF1	0.3713	476.4	-0.1%
SP1	0.4002	1718.3	1.4%
Leading species			
Jack pine	0.4091	658.3	4.7%
White spruce	0.3924	587.2	1.9%
Black spruce	0.3887	478.0	-0.1%
Balsam fir	0.3548	626.3	1.6%
Cedar	0.2683	0.0	0.0%
Larch	0.2926	170.1	4.8%
White birch	0.3292	2399.9	12.8%
Trembling aspen	0.3401	3166.2	5.3%
Leading species within Forest Unit			
MW1 – Aspen	0.3476	754.2	0.0%
MW1 – Birch	0.3898	193.3	0.0%
MW1 – Pine	0.3748	475.8	0.0%
MW1 – Spruce	0.3978	200.0	1.1%
MW2 – Aspen	0.2605	6537.2	0.0%
MW2 – Birch	0.2783	3123.1	10.0%
MW2 – Spruce	0.3573	776.1	0.0%

[7] $$\hat{NMV}_i = \frac{sppba_i \cdot (1 - \hat{cull}_i) \cdot \hat{vbar}_i}{\sum_i sppba_i \cdot (1 - \hat{cull}_i) \cdot \hat{vbar}_i} \cdot \hat{GMV}$$

where $sppba_i$ is the basal area for species i

Results and Discussion

New yield curves

The fit of each model was evaluated using the mean squared error and the mean prediction error expressed as a percentage of the mean (%ME).

The basal area predictions (Eq. [2] and Table 4) were reasonable with %MEs of less than 4% for all forest units and leading species except the MW2. The %ME of the density predictions (Table 5) was less than 5% for all forest units except the SF1 (6.9%) and all leading species except white spruce (9.1%). The %ME of the gross total volume predictions (Table 6) was less than 5% for all forest units except the

Table 7. Parameters and errors associated with the gross merchantable volume model ($\hat{GMV} = z_3 \cdot (1 - e^{-z_1 \cdot DBH_q^{z_2}}) \cdot \hat{GTV}$) by forest unit and leading species

Stratification	z1	z2	z3	Mean squared error	Mean error as % of mean
Forest Unit					
BW1	0.0003	2.9870	0.7111	314.9	-1.9%
LC1	0.1541	0.7754	1.0000	423.3	-5.8%
LH1	0.0612	0.6900	1.0000	972.6	11.5%
MW1	0.1291	0.7515	1.0000	943.5	-0.8%
MW2	0.0001	3.6610	0.6950	2503.8	17.7%
PJ1	0.0000	4.2741	0.8187	297.7	1.7%
PJ2	0.0085	1.9618	0.9023	979.7	2.4%
PO1	0.1312	0.7213	1.0000	1743.9	-5.0%
SB1	0.1094	0.8934	1.0000	532.7	-2.3%
SF1	0.0002	3.3211	0.8820	194.4	0.4%
SP1	0.1576	0.7621	1.0000	1492.8	1.0%
Leading species					
Jack pine	0.1500	0.7840	1.0000	701.6	-2.5%
White spruce	0.0000	3.9370	0.7598	331.2	0.1%
Black spruce	0.1700	0.7128	1.0000	778.2	-2.6%
Balsam fir	0.1000	0.7717	1.0000	777.1	-5.2%
Cedar	0.6500	0.5000	0.5000	1.7	1.2%
Larch	0.0000	3.6397	0.8275	31.2	1.3%
White birch	0.0000	3.8864	0.5843	780.7	10.1%
Trembling aspen	0.0000	3.4521	0.7751	1977.1	-2.8%
Leading species within Forest Unit					
MW1 – Aspen	0.1250	0.7753	1.0000	851.3	-7.0%
MW1 – Birch	0.0024	2.3421	0.7241	134.5	1.7%
MW1 – Pine	0.0079	1.9033	0.8115	240.7	1.2%
MW1 – Spruce	0.0084	1.8234	0.9011	197.9	24.5%
MW2 – Aspen	0.0002	3.4806	0.7417	2771.7	0.0%
MW2 – Birch	0.0001	4.1312	0.5999	988.6	-3.8%
MW2 – Spruce	0.0020	2.2962	0.9562	353.7	0.0%

LC1 (-5.8%), the LH1 (9.2%) and the MW2 (19.7%) and all leading species except white birch (12.8%) and trembling aspen (5.3%). The %ME of the gross merchantable volume predictions (Table 7) was less than 5% for all forest units except the LH1 (11.5%) and the MW2 (17.7%) and all leading species except white birch (10.1%) and balsam fir (5.2%).

In general, the poorest results (based on %ME) were associated with the LH1 and MW2 forest units and the white birch leading species.

Validation and comparison to other models

How good is the growth and yield model presented here? One method of evaluating a model is validation with an independent data set to determine whether the model produces satisfactory predictions in conditions similar to the intended application (Rykiel 1996).

Since 1997, the Growth and Yield Science Unit, a partnership of government and forest industry in Ontario, has established more than 2500 PGPs. These data were withheld from model calibration for the purpose of validating the performance of the models developed using historic and OMNR PSPs. Plots of natural origin located in stands older than 20 years and with a stocking of greater than 0.30 were used (Table 3).

The GTVs predicted using the new yield curves were compared to the observed GTVs by forest unit (Fig. 2). The most obvious prediction problems were for the MW2 forest unit. In Fig. 2e, the forest unit model seriously underpredicted the GTV of softwood-dominated stands. The MW2 forest unit is composed of mixed coniferous–deciduous stands (Table 2) and is often multi-storied. In classifying stands into forest units, MW2 includes all stands not identified as another forest unit. For the SP1 forest unit (Fig. 2k), five PSPs appeared as outliers. Despite being classified as within the SP1 forest unit, these PSPs have jack pine as the leading species; thus, the poor predictions are likely an indication of poor forest unit classification rather than poor model performance. Based on Fig. 2, the predictions using the forest unit stratification were more accurate and precise than those based on the leading species stratification with the exception of the MW2.

The same trends of underpredicting conifer-dominated MW2 plots and jack pine-dominated SP1 plots were observed when prediction errors were plotted by age (Fig. 3). These graphs emphasize the underprediction (Fig. 3) emphasize the underprediction of the leading species stratification for the BW1, MW1, PO1, and SF1 forest units relative to the forest unit stratification.

The MW2 predictions were particularly poor so trends in mixedwood forest units (MW1 and MW2) were further examined by comparing the average prediction error for the forest unit, leading species, and leading species within forest unit predictions (Fig. 4). The leading species within forest unit had the smallest average prediction error for basal area and NMV for the MW1 forest unit, but the leading species stratification had the smallest average prediction error for NMV prediction for the MW2 forest unit. The forest unit or leading species stratifications were superior to the leading species within forest unit stratification so the latter was not investigated further.

The predictions were also compared against the other yield tables in common use in Ontario. In general, Plonski and modified Plonski yield curves overpredicted GTV volume while the forest unit and leading species approaches generally underpredicted volume for the validation data set (Fig. 5). In general, the forest unit approach had the lowest average prediction error for GTV with the exception of the MW2 and SF1 forest units. These forest units are mixed in terms of species composition, light tolerance, and ages. The MW1 is also a mixed species forest unit but tends to be more even-aged. For the MW2 and SF1 forest units, where the forest unit approach performed poorly, the leading species predictions were considerably more accurate. Pothier and Savard's (1998) models also generally underpredicted GTV, particularly for the BW1, MW2 and SF1 forest units. Note that since their model predicts the yields of trees >9 cm DBH, their results were compared against the actual GTV compiled to a 9-cm

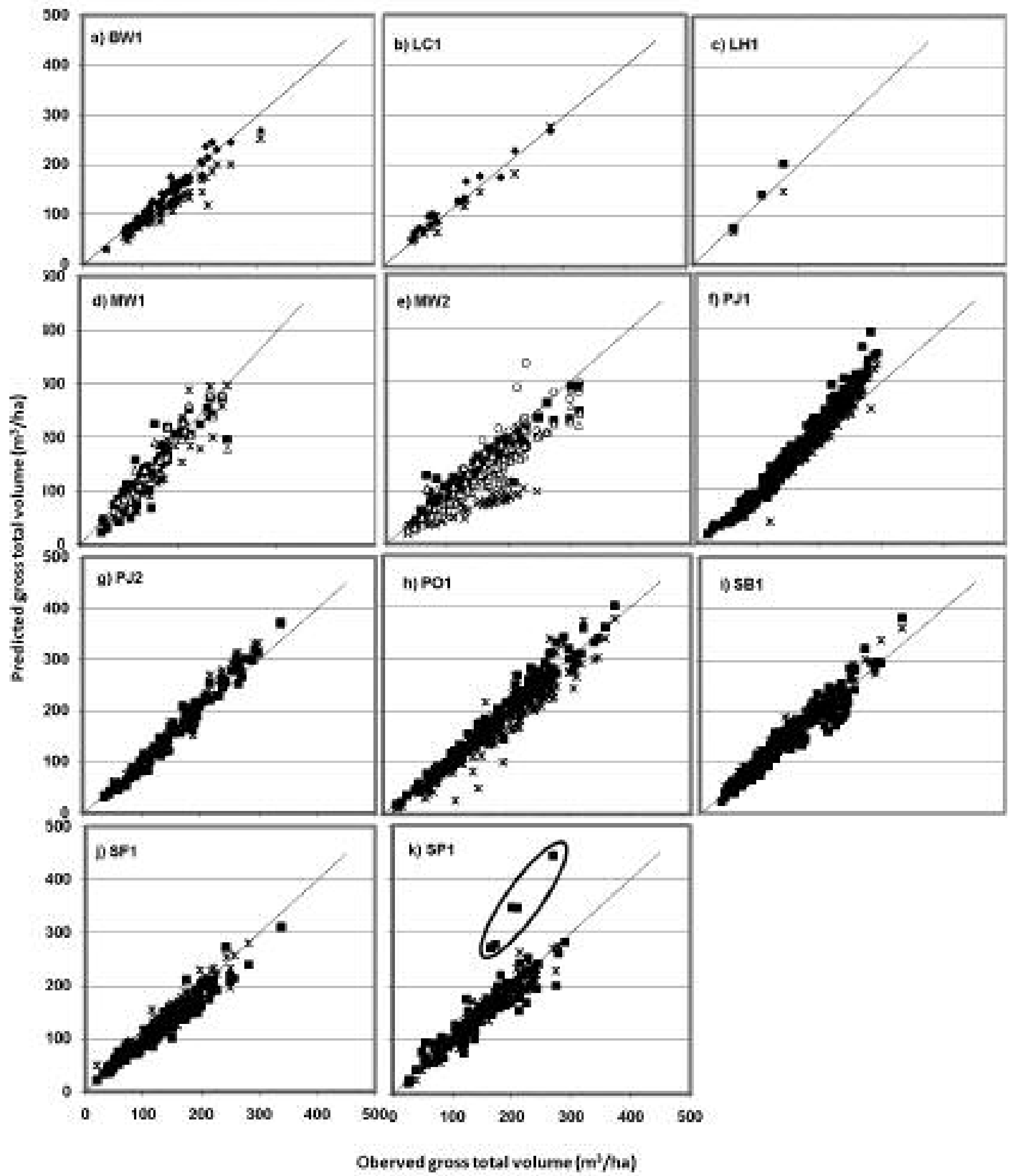

Fig. 2. Gross total volume predictions are plotted against the observed volume for the validation data set by forest unit. ■: forest unit predictions, ×: leading species predictions, and Δ: for the leading species within forest unit for the MW1 and MW2 forest units. In (e) MW2, softwood predictions using the forest unit approach are given by ❍. The circled predictions in (k) SP1 have jack pine as the leading species despite being classified as SP1.

DBH limit.

In general, the average prediction errors for NMV (Fig. 6) were smaller than for GTV due in part to lower volumes. The northeast regional and Plonski modified curves overpredicted the hardwood forest units (BW1, LH1, and PO1). This is likely due to Plonski using a 7-cm small-end diameter merchantability limit compared to the Ontario utilization standard of 16 cm for poplar and birch. The forest unit approach generally produced the lowest mean prediction errors, except for the MW2 and SF1 forest units. The leading species approach led to improved NMV estimates for the MW2 and SF1 forest units but the improvement was not as large as for GTV.

Application

The equations developed here are intended for strategic forest management planning and have been approved for use in

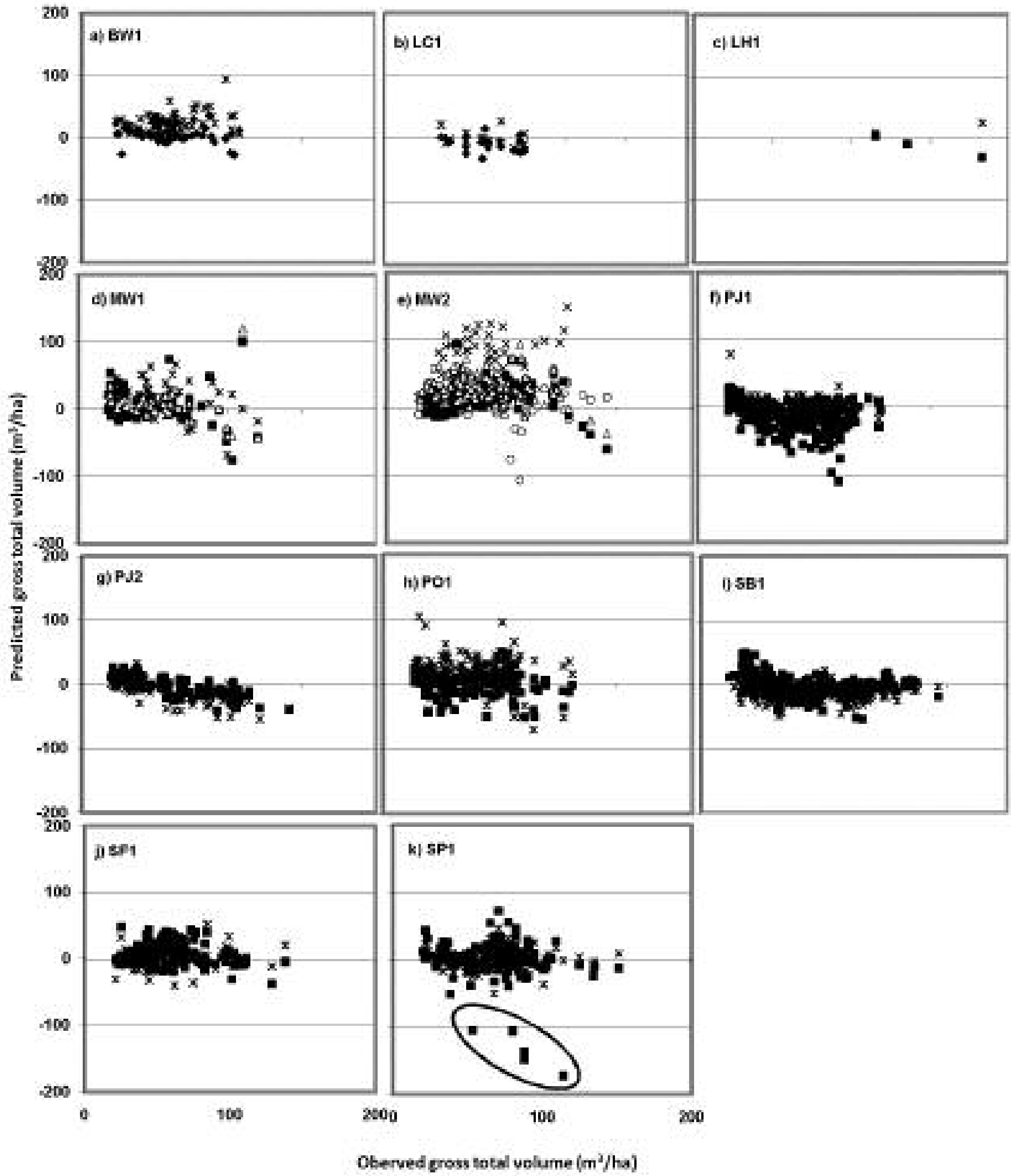

Fig. 3. Gross total volume prediction errors for the validation data plotted by age and forest unit. ■: forest unit predictions, ×: leading species predictions, and Δ: for the leading species within forest unit for the MW1 and MW2 forest units. In (e) MW2 softwood predictions using the forest unit approach are given by ❍. The circled predictions in (k) SP1 have jack pine as the leading species despite being classified as SP1.

Ontario by OMNR. To facilitate their use, a test version of STMMTool that incorporates the new yield curves has been released. SFMMTool takes forest resources inventory and, among many other functions, stratifies the landbase into forest units and produces forest unit yield curves. The user can also specify the outcomes of different intensities of management in terms of species composition, stocking, and average site class or site index and SFMMTool will generate yield tables. A production version of SFMMTool is expected to be released in late 2008.

Conclusion

The yield curves presented here were developed to assist in determining allowable harvest levels during the forest man-

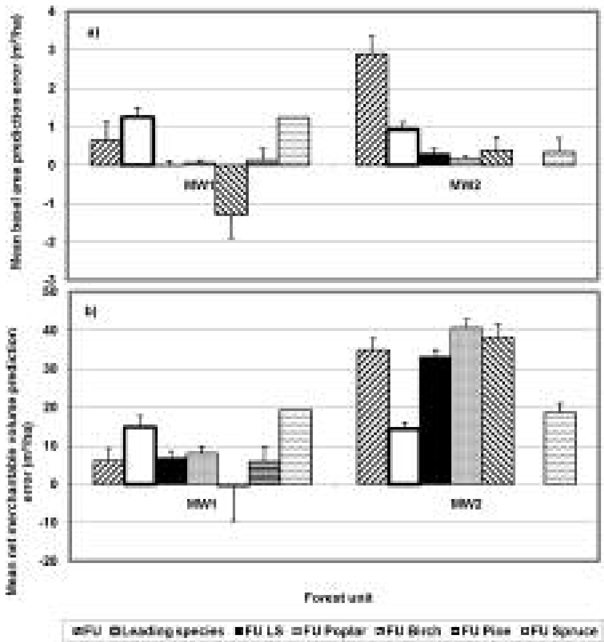

Fig. 4. The mean errors in basal area (a) and net merchantable volume prediction (b) are given for the validation data for the MW1 and MW2 forest units. The stratifications were by forest unit (FU), leading species, and leading species within forest unit (FU LS). Errors for the FU LS stratification are also given by leading species.

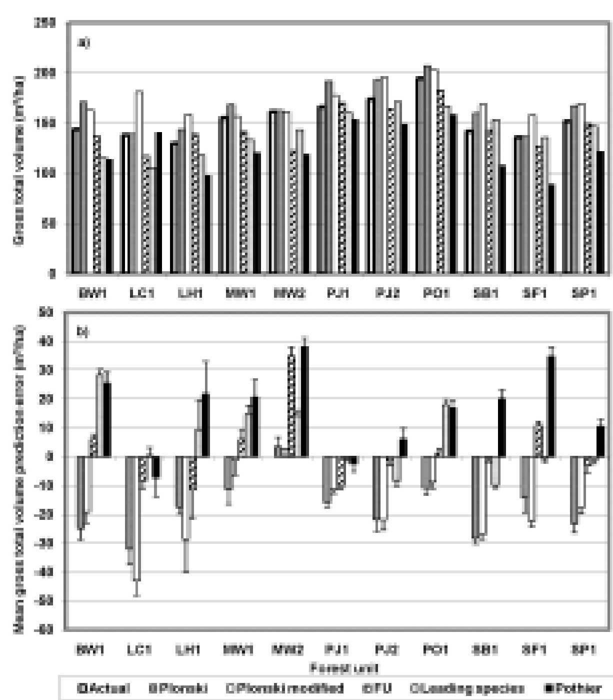

Fig. 5. Mean actual and predicted gross total volume (a) and the mean prediction error (bias) and the standard error of the prediction errors are given by forest unit (b) for the validation data. Gross total volume is not available for the northeast regional yield curves. In contrast to the rest of the models, the Pothier predictions errors are based on gross total volumes for trees >9 cm in diameter at breast height and are compared to the actual volume associated with trees that size. A positive prediction error indicates underprediction.

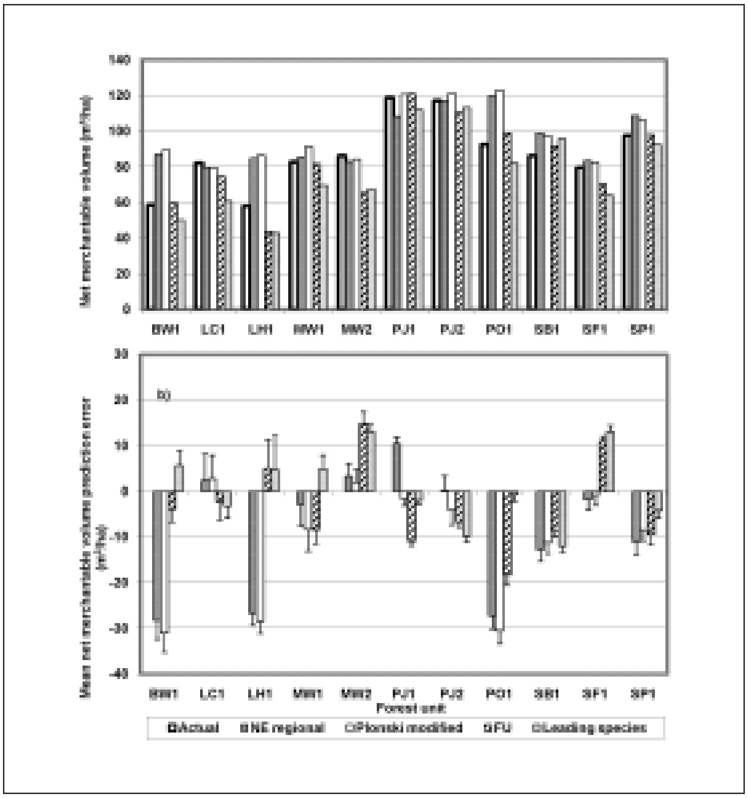

Fig. 6. The average actual and predicted net merchantable volume (a) and the average prediction error (bias) and the standard error of the prediction errors are given by forest unit (b) for the validation data. Net merchantable volume is not available for Plonski's yield curves. A positive prediction error indicates underprediction.

agement planning process. The yield curves are based on substantially more data than existing yield curves for Ontario (Plonski, modified Plonski, and northeast regional curves). They were also developed specifically for the northeast standard forest units. When used to predict yields for an independent data set, the new yield curves were generally more accurate and precise than existing yield curves with the exception of the MW2 and SF1 forest units. These 2 forest units represent a mixture of species, tolerances, ages and yields are not predicted well with the new forest unit-based curves. For these forest units, a leading species approach is recommended.

One reason why the yield curves may not be ideal for some forest unit types is that the FRI site class is not necessarily based on the leading species of the stand. Where possible, future inventories should record the species for which the age, height, and site index were computed to support the development of more accurate yield curves.

The new forest unit-based curves did, however, provide accurate and precise predictions for stand-level planning for forest units with a clear dominant species. The MW2 and SF1 are moister mixedwood forest units and may require leading species models or more detailed tree-level models for increased accuracy. A caution is that the yield curves presented should not be used beyond the range of conditions represented in the data. In particular, low-density plantations, mixed-species plantations, and stands with mid-rotation density regulation (thinning or partial harvesting) require further data collection and/or modelling effort.

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